

Research Papers

Latent heat thermal energy storage in a modular flat plate energy storage with wedge fins

Mohammad Ghalambaz^a, Kourosh Behzadian^b, Antonino La Rocca^c,
Roohollah Babaei-Mahani^d, Amir Keshmiri^{e,f,*}, Pouyan Talebidadehsardari^{f,g}

^a Department of Mathematical Sciences, Saveetha School of Engineering, SIMATS, Chennai, India

^b School of Computing and Engineering, University of West London, London, W5 5RF, United Kingdom

^c Faculty of Engineering, University of Nottingham, Nottingham, United Kingdom

^d Department of Chemical Engineering, Brunel University of London, Uxbridge, UB8 3PH, United Kingdom

^e School of Engineering, University of Manchester, Manchester, M13 9PL, United Kingdom

^f CFD & Engineering Simulation R&D Team, Manchester Simulations Ltd (Mansim), Manchester, UK

^g Power Electronics, Machines and Control (PEMC) Institute, University of Nottingham, Nottingham, United Kingdom

ARTICLE INFO

Keywords:

Phase change material

Energy storage

Modular flat plate

Periodic boundary condition

ABSTRACT

Flat plate latent heat thermal energy storage (LHTES) units can store a notable amount of thermal energy in a compact, modular space, offering efficient energy access through mini-channels. This study aims to investigate the charging and discharging behavior of phase change material (PCM) in a flat plate LHTES unit. Wedge-shaped (triangular) fins were employed to enhance heat transfer on the PCM side. Periodic boundary conditions were applied to account for the modular stacking effect. The impact of fin shape and arrangement on the average charging and discharging power was examined. Furthermore, the influence of the convective heat transfer coefficient in the heat transfer fluid channels and thermal boundary conditions was investigated. The results show that LHTES performance is highly sensitive to fin geometry, boundary conditions, and heat transfer coefficients. A moderate fin height ratio of 0.5 yielded the fastest melting times, outperforming both lower and higher ratios by approximately 8%. Using insulated boundaries instead of periodic ones increased melting times by around 1.6%. Enhancing the convective heat transfer coefficient from 50 to 150 W/m²·K reduced melting duration by 44.9%. Increasing the fin tooth shape ratio from 0.75 to 1.5 led to a 39% reduction in melting time and nearly 50% shorter solidification time. At MVF = 0.7 and equal tooth ratios, top-fin configurations reduced melting time by 12.5–26.2% relative to bottom-only configurations. Configurations without fins required up to 327% longer charging (melting) time than the best-performing configuration; equivalently, the best-performing evaluated case reduced melting time by about 76.6% relative to the finless insulated case, identifying the best case within the tested parameter set rather than a formal optimum. Using the initial-to-target energy change divided by the corresponding elapsed time, increasing the convective coefficient from 50 to 150 W/m²·K for the matched T3/T8 geometry raised average charging power from approximately 0.060 to 0.112 kW/m and average discharging power from approximately 0.055 to 0.086 kW/m.

1. Introduction

Latent heat thermal energy storage (LHTES) systems based on phase change materials (PCMs) have attracted increasing attention because they can store and release large amounts of thermal energy over a relatively narrow temperature range. During charging, the PCM absorbs

thermal energy as it melts, while during discharging it releases the stored latent heat as it solidifies. This nearly isothermal behavior is particularly useful when thermal energy must be stored compactly and delivered at a controlled temperature. A recent review by Arévalo et al. [1] emphasized the importance of thermal energy storage in renewable-energy systems, while Carnie et al. [2] demonstrated the relevance of inter-seasonal PCM storage for decarbonizing building heating and

* Corresponding author at: School of Engineering, University of Manchester, Manchester, M13 9PL, United Kingdom.

E-mail addresses: m.ghalambaz@gmail.com (M. Ghalambaz), kourosh.behzadian@uwl.ac.uk (K. Behzadian), antonino.larocca@nottingham.ac.uk (A. La Rocca), roohollah.babaei-mahani@brunel.ac.uk (R. Babaei-Mahani), A.Keshmiri@manchester.ac.uk (A. Keshmiri), pouyan.talebizadehsardari2@nottingham.ac.uk (P. Talebidadehsardari).

<https://doi.org/10.1016/j.est.2026.124486>

Received 25 March 2026; Received in revised form 13 August 2026; Accepted 28 August 2026

2352-152X/© 2026 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY-NC license (<http://creativecommons.org/licenses/by-nc/4.0/>).

Symbols			
$\partial / \partial t, \partial / \partial x, \partial / \partial y$	Partial derivatives with respect to time and space, –	E_{released}	Positive energy decrease from the physical initial state to MVF = 0.1 per unit depth, J/m
A_{mush}	Mushy zone constant (Momentum sink factor), $\text{kg}/(\text{m}^3 \cdot \text{s})$	NR_f	Fin-height ratio, –
A_μ	Artificial viscosity ($1\text{E4 Pa}\cdot\text{s}$), $\text{Pa}\cdot\text{s}$	NR_t	Tooth aspect ratio, –
C_p	Specific heat capacity, $\text{J}/(\text{kg}\cdot\text{K})$	P_{ch}^-	Average charging power per unit depth, kW/m
f	Liquid fraction (function of temperature), –	P_{dis}^-	Average discharging power per unit depth, kW/m
g	Gravitational acceleration, m/s^2	Q_0	Total thermal-energy state quantity at physical $t = 0$, J/m
h_{conv}	Convective heat transfer coefficient, $\text{W}/(\text{m}^2 \cdot \text{K})$	$t_{\text{MVF}=X}$	Elapsed time from physical $t = 0$ to target MVF X , s
k	Thermal conductivity of PCM, $\text{W}/(\text{m}\cdot\text{K})$	Subscripts	
L	Latent heat of PCM, J/kg	0	Physical initial state
MVF	Melt volume fraction, –	ch	Charging
p	Pressure, Pa	dis	Discharging
q	Heat flux, W/m^2	f	Fusion temperature or fin-height ratio, as indicated by context
Q_{latent}	Latent heat, J/m	latent	Latent thermal-energy contribution
Q_{sensible}	Sensible heat, J/m	m	Melting
Q_{total}	Total sensible-plus-latent thermal-energy state quantity, J/m	MVF	Target melt volume fraction
T	Temperature, K	mush	Mushy-zone quantity
t	Time, s	released	Released-energy change
T_0	Reference temperature or initial temperature, K	sensible	Sensible thermal-energy contribution
T_f	Fusion temperature, K	stored	Stored-energy change
u, v	Velocity components in x and y, m/s	t	Time-saving metric or tooth aspect ratio, as indicated by context
V	Volume of PCM domain, m^3	conv	Convection boundary conditions at the channel side
β	Heat volume expansion coefficient, $1/\text{K}$	dst	Destination side (for periodic boundary conditions)
η_t	normalized time-saving percentage, –	fin	Fins and solid walls
λ_{mush}	Mushy zone permeability, –	l	Liquid PCM
μ	Dynamic viscosity of PCM, $\text{Pa}\cdot\text{s}$	PCM	Phase change material
Π	Normalized phase change time, –	s	Solid PCM or solidification
ρ	Density of phase change material, kg/m^3	src	Source side (for periodic boundary conditions)
E_{stored}	Positive energy increase from the physical initial state to MVF = 0.9 per unit depth, J/m	total	Total quantity (e.g., sensible-plus-latent energy or total fin length)

cooling. These applications highlight the potential of LHTES systems for solar thermal systems, building energy management, waste-heat recovery, and compact thermal management.

The main advantage of PCMs is their high energy storage density compared with sensible heat storage materials. This allows LHTES units to store a relatively large amount of energy in compact volumes, which is important for modular and space-limited applications. However, many commonly used PCMs have low thermal conductivity, which limits the rate of heat transfer during both melting and solidification. Sarbu and Dorca [3] reviewed this limitation in latent heat storage systems and showed that the low thermal conductivity of PCMs can significantly slow charging and discharging. Yang et al. [4] also identified poor heat transfer as one of the main barriers to the practical implementation of PCM-based storage systems. Therefore, improving heat transfer inside the PCM domain remains a central design challenge for LHTES systems.

Several enhancement strategies have been proposed to overcome this limitation, including high-conductivity additives, metal foams, nano-enhanced PCMs, encapsulation, cascaded PCM arrangements, and extended surfaces. Hordov et al. [5] reviewed organic and inorganic PCM composites and discussed how material selection affects thermal storage performance. Ghalambaz et al. [6] showed that nano-encapsulated PCMs have become an active research topic because they can improve thermal response and system integration. Kabasa et al. [7] reviewed cascaded PCM storage systems and highlighted the need for optimized design under realistic operating conditions. Dedicated finned tubes have also been shown to improve charging and discharging kinetics in shell-and-multi-tube LHTES systems for building applications [8]. Although these techniques are promising, fins remain one of the

most practical passive enhancement methods because they are simple, mechanically robust, and directly connected to the heat-transfer surface.

Fins improve LHTES performance by increasing the effective heat-transfer area and creating conductive pathways from the heated or cooled wall into the PCM. Al-Salami et al. [9] recently reviewed fin-enhanced PCM charging and showed that fin geometry, fin number, and fin distribution strongly influence the melting process. Other studies have also shown that the number, length, thickness, and flexibility of fins can significantly affect heat transfer performance [10]. In a recent optimization study of a finned rectangular LHTES unit, response surface methodology and the Taguchi technique were used to examine the effects of fin arrangement, fin angle, chamber aspect ratio, and fin position on the melting process [11]. The results showed that a symmetric fin configuration with a larger fin surface area near the lower part of the chamber could shorten the melting time by up to 57%, while the optimized design increased the heat storage rate by about 15% compared with the reference case. For example, modified fin shapes such as curved fins, wavy fins, pin fins, perforated fins, and biomimetic fins have been investigated to improve thermal penetration into the PCM domain. Curved fins were shown to reduce melting time by modifying both the conduction path and the flow structure inside the molten PCM [12]. Tree-shaped fins have also been proposed to distribute heat more effectively within the PCM volume [13]. These studies show that fin performance depends not only on the amount of high-conductivity material, but also on where and how this material is distributed.

The role of fins is not identical during melting and solidification. During melting, the process initially starts with conduction because the PCM is solid. As the liquid region grows, buoyancy-driven convection develops and can help redistribute heat within the molten PCM.

Therefore, in the later stages of melting, the relative importance of local fin geometry may decrease because natural convection contributes to heat redistribution. During solidification, the PCM is initially liquid and natural convection may be important at the beginning. However, as the solid layer grows, the remaining liquid region becomes confined, velocity decreases, and conduction becomes increasingly dominant. Kamkari and Shokouhmand [14] experimentally showed that the melting enhancement ratio of partial fins decreases with time after reaching a maximum, indicating that fins are especially effective during the earlier stages of melting. Abdi et al. [15] reported a stronger enhancing effect of fins during solidification than melting, with mean-power improvements of 395% and 90%, respectively. Recent triplex-tube results confirm this behavior: optimized staggered circular fins reduced solidification time by 52% and increased discharge rate by 30%, showing fins are crucial when conduction controls cooling [16].

Flat-plate and plate-type LHTES systems are attractive because they provide compact structure, large heat-transfer area, and convenient integration with heat-transfer-fluid (HTF) channels. In these systems, PCM is typically placed between plate-like heat-transfer surfaces, while the HTF flows through neighboring channels to charge or discharge the storage unit. A study on corrugated plate heat exchanger LHTES systems showed that modifying the plate geometry can significantly reduce the complete melting and solidification times compared with conventional cylindrical storage systems [17]. Experimental work on pillow-plate heat exchangers also demonstrated that compact plate-type structures can provide good heat transfer rates during PCM charging and discharging [18]. Another numerical investigation showed that plate thickness and HTF operating conditions strongly affect the thermal response of flat-plate LHTES modules [19]. These results confirm that plate-type geometries are promising candidates for modular thermal storage.

Recent work has further highlighted the relevance of plate and flat-plate PCM storage. Taghavi et al. [20] performed a multi-objective optimization of a plate heat exchanger thermal energy storage system with PCM and showed that the dimensions of the PCM and HTF sections strongly affect the compromise between stored energy and thermal power. Wang and Liu [21] investigated a multiple flat-plate latent heat storage heat exchanger and reported that the arrangement of the plates has a strong influence on thermal performance. These recent studies confirm that plate-type PCM heat exchangers remain an active research direction. However, most available studies focus on heat transfer along the HTF flow direction or model the full HTF channel geometry. Less attention has been given to cross-sectional periodic models that represent repeated cells inside a modular flat-plate stack.

Periodic modeling is useful when a device consists of repeated geometrical units with similar thermal and flow conditions. Instead of simulating the full stack, an interior representative cell can be modeled by applying periodic thermal and momentum boundary conditions. This approach can reduce computational cost while preserving the influence of neighboring cells. It is particularly relevant for modular flat-plate LHTES systems because identical PCM–fin–channel units are repeated next to each other. However, the validity of periodic boundary conditions depends on the physical structure of the module. Periodic boundaries are representative of internal cells away from external edges, manifolds, casing effects, and inlet/outlet non-uniformities. In contrast, if neighboring teeth or surfaces are separated by insulating spacers, gaskets, rubber layers, or air gaps, an adiabatic boundary may be more appropriate. Therefore, comparing periodic and insulated boundary conditions is necessary for understanding the modular stacking effect.

In addition to boundary conditions, the shape of the fins on the PCM side and the tooth geometry on the HTF side can strongly affect the thermal behavior of the storage unit. Conventional straight fins may not distribute heat effectively when the amount of high-conductivity material is fixed. Wedge-shaped or triangular fins provide an alternative distribution of copper, with a wider base near the heat-transfer wall and a tapered extension into the PCM. This geometry can reduce thermal

resistance near the wall while allowing heat to penetrate deeper into the PCM. At the same time, the HTF-side tooth geometry influences the available convective surface and the way heat is transferred from the HTF-side boundary into the copper structure. Therefore, the combined effects of PCM-side wedge fins, HTF-side tooth shape, fin height ratio, fin placement, and boundary condition should be examined systematically.

Despite the progress in PCM heat exchangers and fin-enhanced LHTES systems, several gaps remain. First, modular flat-plate LHTES units with repeated wedge-fin cells have not been sufficiently investigated. Second, the interaction between fin distribution and periodic or insulated boundary conditions has not been clarified for both charging and discharging. Third, many previous studies report global melting time or stored energy, but provide less discussion of how phase-front morphology, conductive pathways, and natural convection change with geometric parameters. These gaps are important because the best configuration for melting is not necessarily the best configuration for solidification.

The present study addresses these gaps by investigating phase-change heat transfer in a modular flat-plate LHTES unit equipped with wedge-shaped fins. A two-dimensional periodic model is developed across a section perpendicular to the HTF flow direction. The HTF channel is not geometrically resolved; instead, its effect is represented through a prescribed convective boundary condition at the channel-side surfaces. The PCM domain, copper walls, and PCM-side wedge fins are solved explicitly using the finite element method coupled with the enthalpy-porosity approach. The effects of fin height ratio, tooth shape ratio, fin placement, convective heat transfer coefficient, and periodic or insulated boundary conditions are systematically examined during both melting and solidification. The objective is to clarify how wedge-fin geometry and modular boundary assumptions influence phase-change times, initial-to-target stored and released energy changes, average charging and discharging power, phase-front evolution, and natural-convection behavior in flat-plate LHTES modules.

2. Model and governing equations

2.1. Model description

The conceptual framework of a concentrated solar heating system designed for building applications highlights the pivotal role of Latent Heat Thermal Energy Storage (LHTES) in advancing solar energy technologies. This relevance stems from the inherently intermittent nature of solar energy, which is influenced by both temporal and meteorological factors. LHTES systems offer the advantage of storing significant thermal energy at phase change temperatures within compact volumes, thereby mitigating the variability typically observed in transitional solar systems.

The proposed system comprises a solar collector, a storage tank, an LHTES module, and a series of circulation pumps. The collector elevates the temperature of water, which is then retained in a hot water storage tank as sensible heat to meet the building's heating demand. Additionally, an auxiliary thermal loop connects the hot water tank to the LHTES unit. When there is either surplus energy production or reduced thermal demand, the pump in the auxiliary circuit is activated to transfer excess heat into the LHTES module. Conversely, during periods of insufficient solar energy, thermal energy stored in the LHTES unit is extracted to compensate for the deficit. A schematic representation of this system configuration is illustrated in Fig. 1.

Fig. 2 presents the design of a LHTES device in a flat plate energy storage device (Fig. 2(a)). The device is made of HTF channels and PCM blocks. The copper fins are made of two shapes. A tooth shape part, which makes the HTF channel, and a wedge part which extended into the PCM and increases the heat transfer rate in contact to the PCM. The plates are modular and several numbers of modules can be placed on top of each other (Fig. 2(b)). The casing of a module provides passages to transport and direct the water flow between the modules and into HTF

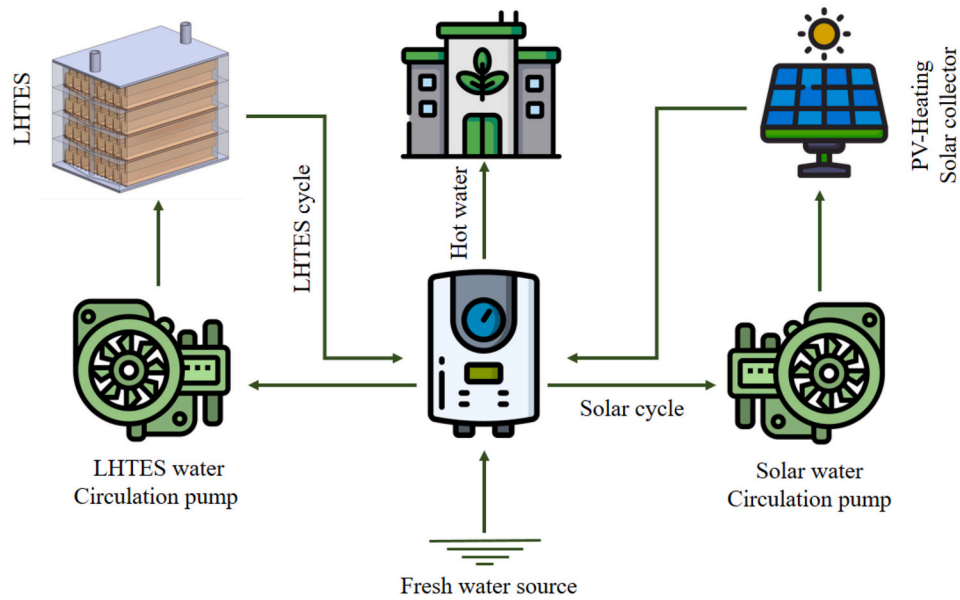


Fig. 1. Schematic representation of the integrated solar-assisted heating system incorporating a LHTES unit.

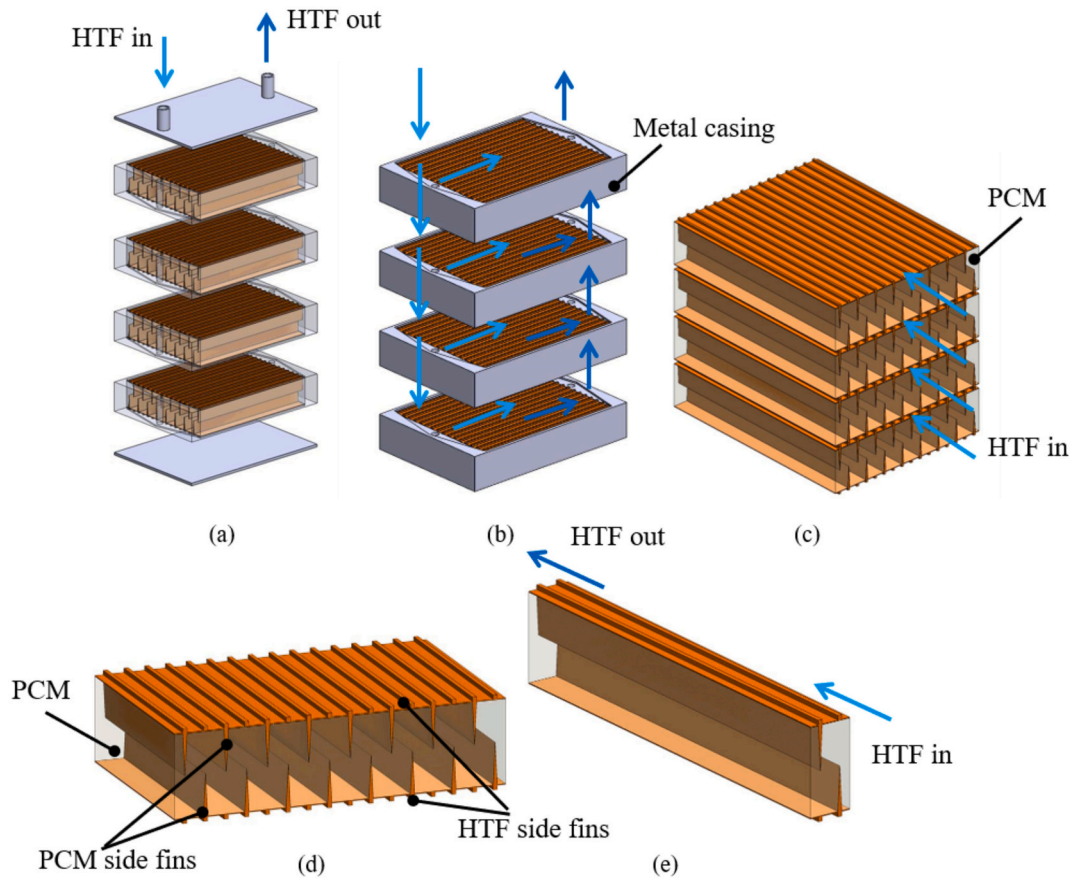


Fig. 2. Exploded and isometric schematic representations of the LHTES unit: (a) layered exploded view showing HTF inlet/outlet configuration, (b) staged assembly with directional flow arrows indicating HTF flow paths, (c) perspective of the final packed structure with integrated PCM and HTF channels, (d) cutaway view detailing the placement of PCM side fins and HTF channel fins, (e) schematic of a single rectangular HTF channel with inlet and outlet boundaries.

channels. Fig. 2(c) shows a transparent view of the unit and the HTF passages. A single module in transparent shape is depicted in Fig. 2(d). The fins and channels repeat next to each other. Thus, a single passage and the PCM domain below the passage (Fig. 2(e)) can represent the

behavior of a module. In this case, the periodic boundary condition should be applied to link the flow and heat transfer behavior of left and right sides of the selected domain. Fig. 3(a) shows a 2D view of the unit where the tooth from two modules have created the channel passages.

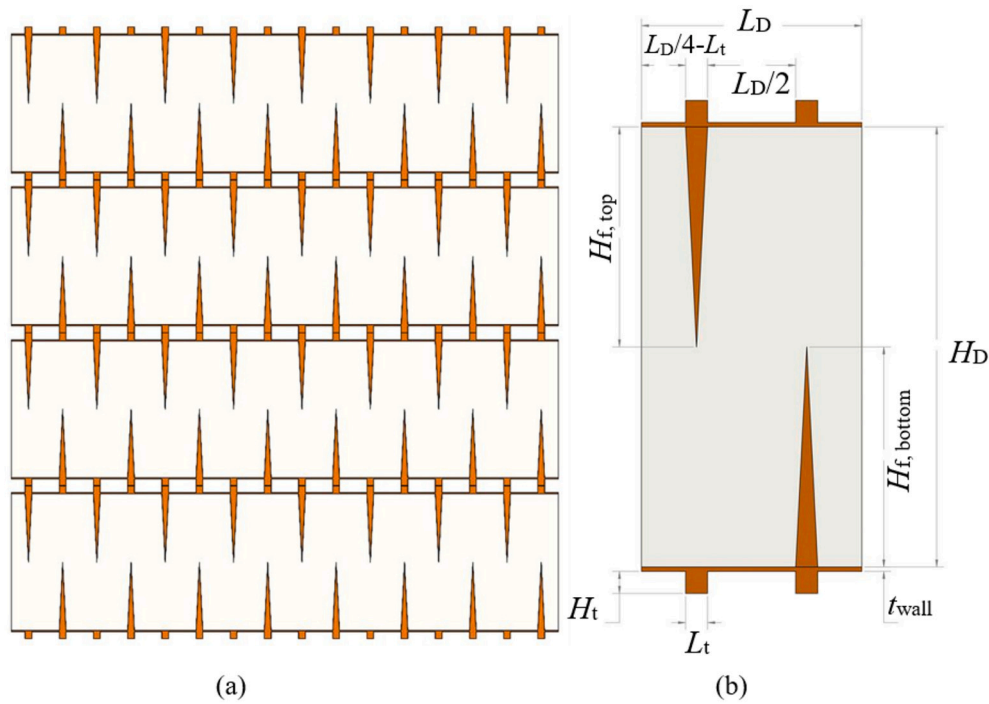


Fig. 3. Geometric layout and dimensional parameters of the enhanced LHTES module: (a) 2D planar configuration of repeated modular cells, and (b) annotated cross-sectional schematic of the periodic computational cell. Here, H_D denotes the clear PCM-region height, while the HTF channel is represented through the convective boundary condition rather than being geometrically resolved.

Fig. 3(b) depicts the geometrical details of the periodic domain (see Fig. 2(e)). The applied boundary conditions on the model are illustrated in Fig. 4. The side borders of the model create a pair of periodic conditions. The top and bottom teeth can create periodic or insulated boundary conditions. If the copper teeth of neighboring modules are in close direct contact, the corresponding tooth boundaries can be considered periodic. If an insulating layer, sealing rubber, gasket, or separated interface exists between neighboring tooth surfaces, these boundaries are treated as insulated. The remaining area of the channels and the side of the tooth is subject to convective heat transfer with the HTF flow and is modeled with a prescribed temperature (T_{conv}) and convective heat transfer coefficient (h_{conv}).

HTF conveys thermal energy to the channel walls and fin surfaces primarily through conduction. Subsequently, the heat is conducted from the channel walls and extended fins into the PCM region. During the phase change process, the PCM absorbs or releases latent heat at a defined fusion temperature, T_f . Depending on whether the system is undergoing the charging or discharging phase, thermal conduction through the tube wall causes the water to transfer heat to or from the paraffin wax PCM, leading to a corresponding increase or decrease in the HTF outlet temperature.

To meet the thermal capacity and operational demand specified by the cycle illustrated in Fig. 1, multiple LHTES units can be configured in parallel, in series, or through a combination of both arrangements. The core objective of the current study is to investigate the heat transfer characteristics within an LHTES unit that integrates a PCM domain with improved wedge-shaped fins and rectangular HTF flow channels.

At the onset of the charging process, when the LHTES unit is at a relatively low temperature ($T_c < T_h$), the hot storage tank, maintained at a constant elevated temperature T_h , supplies excess thermal energy to the LHTES unit. Accordingly, T_h is the inlet temperature of the HTF during the charging phase.

The copper assigned to the HTF-side teeth and PCM-side wedge fins is kept constant for the finned cases T1–T24; T25 and T26 intentionally omit the PCM-side wedge fins and are excluded from this constraint. The tooth height can be increased which instead reduces the tooth width.

The computational domain represents one periodically repeated cross-sectional unit of the modular flat-plate LHTES module in a plane perpendicular to the HTF flow direction. The base dimensions of the selected periodic cell are $L_D = 50$ mm and $H_D = 2L_D = 100$ mm, which define the width and height of the PCM-region envelope, respectively. The copper wall thickness separating the PCM from the HTF-side channel surface is $t_{wall} = 1$ mm. The HTF-side tooth geometry is defined by $L_t = 5/NR_t$ mm and $H_t = 5NR_t$ mm, so that $L_t H_t = 25$ mm² remains constant when NR_t is varied. The total wedge-fin length in each periodic cell is $H_{f,total} = H_{f,top} + H_{f,bottom} = 20H_t$. The fin-height ratio is defined as $NR_f = H_{f,top}/H_{f,total}$, giving $H_{f,top} = NR_f H_{f,total}$ and $H_{f,bottom} = (1 - NR_f) H_{f,total}$. Therefore, within the finned comparisons, the total copper amount assigned to the tooth and wedge fins is kept constant while the tooth aspect ratio and fin distribution are varied. Within the same outer envelope, the finless T25 and T26 cases contain less copper and more PCM and are used only as device-level fin/no-fin baselines. For the reference case with $NR_t = 1.0$ and $NR_f = 0.5$, $L_t = 5$ mm, $H_t = 5$ mm, $H_{f,total} = 100$ mm, and $H_{f,top} = H_{f,bottom} = 50$ mm.

The HTF channel flow is not solved explicitly in the present model. Instead, the effect of the HTF-side mini-channel is represented by a convective boundary condition with prescribed T_{con} and h_{con} on the exposed channel-wall and tooth-side surfaces. Thus, the present model resolves the PCM region, copper walls, and PCM-side wedge fins, while the channel-side heat transfer is incorporated through h_{con} . Since the model is two-dimensional, the results are reported per unit depth in the HTF flow direction. Table 1 presents the operational temperature range and the detailed thermophysical characteristics of materials used in the LHTES unit, including paraffin as the PCM, copper as the fin material, and water as the heat transfer fluid.

2.2. Governing equations

The governing equations for the copper fins, channel walls, and the PCM region are formulated through partial differential equations. Since all structural components aside from the PCM are composed of copper, the term “fin” with the subscript f is used hereafter to represent both the

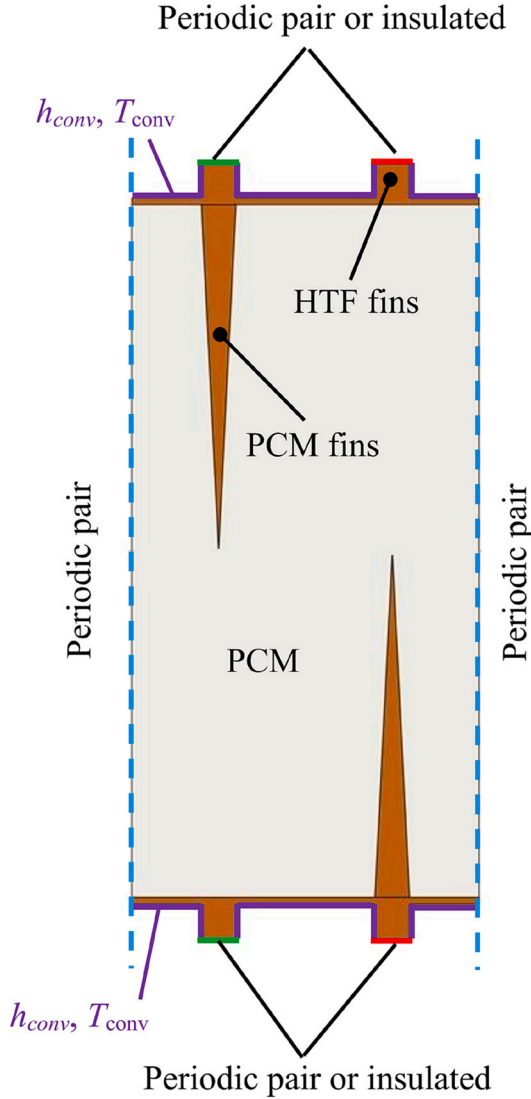


Fig. 4. Boundary condition and symmetry considerations for the computational domain.

fins and the channel walls. In these solid copper regions, heat transfer occurs via conduction, whereas natural convection is assumed in the liquid phase of the PCM.

To simulate the phase change behavior of PCM, the enthalpy-porosity technique is employed. This method incorporates additional source terms into the momentum equations to enforce zero velocity in regions of fully solidified PCM, governed by the liquid fraction function (f). Due to the relatively small temperature gradients in the studied configuration, thermophysical properties are considered constant across most materials, except for PCM, which exhibits temperature-dependent properties during the melting process.

The complete nomenclature and thermophysical parameters used in the mathematical model are summarized in Table 1. The corresponding conservation equations for mass, momentum, and energy are expressed

in partial differential form [27–29]:

Continuity:

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (1)$$

Momentum:

$$\rho_{PCM} \frac{\partial u}{\partial t} + \rho_{PCM} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \left(\frac{\partial p}{\partial x} \right) + \left[\frac{\partial}{\partial x} \left(\mu_{PCM} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{PCM} \frac{\partial u}{\partial y} \right) \right] - A_{mush} \frac{(1-f)^2}{\lambda_{mush} + f^3} u \quad (2)$$

$$\rho_{PCM} \frac{\partial v}{\partial t} + \rho_{PCM} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \left(\frac{\partial p}{\partial y} \right) + \left[\frac{\partial}{\partial x} \left(\mu_{PCM} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{PCM} \frac{\partial v}{\partial y} \right) \right] + g \rho_{PCM} \beta_{PCM} (T_{PCM} - T_0) - A_{mush} \frac{(1-f)^2}{\lambda_{mush} + f^3} v \quad (3)$$

Conservation of energy in the PCM:

$$(\rho C_p)_{PCM} \frac{\partial T_{PCM}}{\partial t} + (\rho C_p)_{PCM} \left(u \frac{\partial T_{PCM}}{\partial x} + v \frac{\partial T_{PCM}}{\partial y} \right) = \frac{\partial}{\partial x} \left(k_{PCM} \frac{\partial T_{PCM}}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_{PCM} \frac{\partial T_{PCM}}{\partial y} \right) - \rho_{PCM} L_{PCM} \frac{df(T_{PCM})}{dt} \quad (4)$$

The latent heat of fusion (L), heat volume expansion coefficient (β), and the gravitational acceleration (g) are incorporated into the governing equations discussed above. Subscripts are used throughout the formulation to denote PCM.

To constrain fluid motion within the solid regions of the PCM domain, specific source terms, namely A_{mush} (set to 10^6 Pa·s/m²) and λ_{mush} (assigned a value of 0.001), are employed. The large magnitude of A_{mush} effectively suppresses velocity in the solid zones by dominating the momentum equations, while the small value of λ_{mush} ensures numerical stability and prevents singularities within the source term formulation.

To further promote numerical stability and accurately represent flow behavior in the mushy region, a viscosity model dependent on the liquid fraction (f) is adopted. In this model, dynamic viscosity is expressed as $\mu = A\mu(1-f) + \mu_{PCM,l}f$, where $A\mu$ represents a large viscosity value (on the order of 10^4 Pa·s) associated with the solid phase [30,31]. This formulation ensures that as $f \rightarrow 1$, the viscosity approximates that of the liquid PCM ($\mu_{PCM,l}$), while at $f = 0$, the viscosity becomes substantially higher, mimicking the resistance to flow in solid PCM. It should be noted that the artificial viscosity follows the phase behavior and only smooths the changes in momentum gradients across the mushy region, thereby improving convergence and the stability of the equations. This term loses its effect as the velocity gradients drop in the solid and mushy regions. Thus, this term quickly diminishes in the mushy and solid regions, where the velocity gradients are small. It also vanishes in the fully liquid region, where $f = 1$. Thus, the effect of this term is merely to improve the stability of the finite element solution for the momentum equations rather than to impact the actual physical behavior of the governing equations. The heat transfer behavior of the PCM domain is governed by the energy equations presented in Eq. (4), while the temperature-dependent definition of the liquid fraction (f) is given by the following relation [28].

Table 1

Thermophysical properties and operating temperature range of materials employed in LHTEs system, including water, paraffin, and copper.

Materials	T_i (K)	L (kJ/kg)	β (1/K)	μ (kg/m.s)	C_p (J/kg.K)	ρ (kg/m ³)	k (W/m.K)
Water [22]	–	–	0.00021	0.000957	4179	997.1	0.613
Paraffin (solid/liquid) [23–25]	322–327	176	0.00091	0.0036	2700/2900	916/790	0.21/0.12
Copper [26]	–	–	–	–	386	8900	380

$$f(T_{PCM}) = \begin{cases} 0 & T < -\frac{1}{2}\Delta T_f + T_f(\text{Solid}) \\ \frac{1}{2} + \frac{(T_{PCM} - T_f)}{\Delta T_f} & -\frac{1}{2}\Delta T_f + T_f \leq T \leq \frac{1}{2}\Delta T_f + T_f(\text{Mixture}) \\ 1 & T > \frac{1}{2}\Delta T_f + T_f(\text{Liquid}) \end{cases} \quad (5)$$

where $\Delta T_f = T_l - T_s$ is the temperature difference between solidus temperature and liquidus temperatures. As shown in the following equations, the thermophysical characteristics of the PCM are calculated using a linearly weighted averaging method.

$$(\rho C_p)_{PCM} = (1-f)(\rho C_p)_l + f(\rho C_p)_s \quad (6)$$

$$\rho_{PCM} = (1-f)\rho_l + f\rho_s \quad (7)$$

The linear interpolation in Eqs. (5)–(7) is applied only within the finite mushy temperature interval, $T_s \leq T \leq T_l$. Outside this interval, the PCM properties correspond to the pure solid or pure liquid state. Therefore, the interpolation is used to smooth the thermophysical-property transition across the phase-change range and to avoid discontinuities in the nonlinear FEM solution. Since the MVF is obtained by integrating the interpolated liquid fraction f over the PCM domain, the mesh convergence of MVF provides a direct measure of the numerical uncertainty associated with the phase-fraction interpolation and domain integration. Within this framework, the subscripts “s” and “l” are used to denote the solid and liquid phases of the PCM, respectively. A summary of the thermophysical properties for both the copper tube and paraffin, PCM, is provided in Table 1. The following equations describe energy conservation within the fins and channel walls.

$$(\rho C_p)_{fin} \frac{\partial T_{fin}}{\partial t} = k_{fin} \left(\frac{\partial^2 T_{fin}}{\partial x^2} + \frac{\partial^2 T_{fin}}{\partial y^2} \right) \quad (8)$$

2.3. The boundaries and initial conditions

The periodic boundary condition is used to represent an interior repeating cell of the modular flat-plate LHTES stack. In the actual device, identical PCM–fin–channel units are repeated laterally and vertically; therefore, an interior cell exchanges heat and momentum with neighboring cells in a repeating manner. The left and right boundaries of the computational domain are treated as periodic pairs to account for the influence of adjacent repeated passages without modeling the full module width. Similarly, the top and bottom tooth boundaries are treated as periodic when neighboring copper teeth are assumed to be in direct thermal contact. When an insulating spacer, gasket, rubber layer, or separated interface exists between the teeth, the top/bottom boundary is modeled as adiabatic. Therefore, the periodic model is representative of the internal region of a sufficiently repeated modular stack, away from external edges, manifolds, inlet/outlet non-uniformities, and casing/end effects.

The HTF (T_{conv}) was set to a high temperature of $T_{conv} = T_f + 15^\circ\text{C}$. The side walls are treated as periodic boundary condition (B.C.). The initial temperature for the solidification was $T_0 = T_f + 15^\circ\text{C}$ while for the melting process it was set as $T_0 = T_l - 15^\circ\text{C}$. The periodic B.C. for the copper in the walls was employed:

$$-n_{src} \cdot q_{src} = n_{dst} q_{dst}, \quad T_{src} = T_{dst} \quad (9)$$

where the continuity of heat flux (q) and temperature between periodic pairs of sources (src) and destination (dst) were applied. A similar periodic condition was also applied for the top and bottom of the tooth for periodic tooth case. In the case of insulated tooth, the zero-heat flux was applied at the top of each tooth. The convective heat transfer at the channel walls and tooth sides was applied as:

$$-n \cdot q = h_{conv}(T_{conv} - T) \quad (10)$$

The no-slip and no-permeability ($u = v = 0$) was applied on all wall surfaces. The continuity of heat and temperature was applied at the copper – PCM interfaces. The B.C. for the temperature, velocities, and pressure on the PCM domain at the periodic B.C.s. was employed as:

$$-n_{src,PCM} \cdot q_{src,PCM} = n_{dst} q_{dst,PCM}, \quad T_{src,PCM} = T_{dst,PCM} \quad (11)$$

$$u_{src,PCM} = u_{dst,PCM}, \quad v_{src,PCM} = v_{dst,PCM}, \quad p_{src,PCM} = p_{dst,PCM} \quad (12)$$

A relative zero pressure reference was considered at the edge of bottom fin.

2.4. Parameters of interest

The extent of phase change in the PCM is quantified by the melted volume fraction, which is evaluated using the subsequent expression:

$$MVF = \frac{\oint_V f dV}{\oint_V dV} \quad (13)$$

In this formulation, dV denotes the differential volume element within the shell region. The system's total sensible and latent thermal-energy state quantity can be determined through the following expressions:

$$Q_{sensible} = \left[\oint_V \left(\int_{T_0}^T (\rho C_p)_{PCM}(T) dT \right) dV \right] + \frac{(T - T_0)(\rho C_p)_f V_f}{f} \quad (14)$$

$$Q_{latent} = \oint_V \rho_{PCM} L_{PCM} f dV \quad (15)$$

The total thermal-energy state quantity, denoted as Q_{total} , is obtained by summing the contributions from both sensible and latent heat:

$$Q_{total} = Q_{sensible} + Q_{latent} \quad (16a)$$

Let $Q_0 = Q_{total}(t = 0)$, then The positive initial-to-target energy changes were calculated first; the corresponding average charging and discharging powers were then obtained by dividing those energy changes by the elapsed times to the target states:

$$E_{stored} = Q_{total}(MVF = 0.9) - Q_0; P_{ch} = E_{stored} / (1000 t_{MVF=0.9}) \quad (16b)$$

$$E_{released} = Q_0 - Q_{total}(MVF = 0.1); P_{dis} = E_{released} / (1000 t_{MVF=0.1}) \quad (16c)$$

Here Q_{total} , E_{stored} , and $E_{released}$ are in J/m; $t_{MVF=X}$ is the elapsed time in s from the physical $t = 0$ state; and dividing each energy change by its elapsed time gives average power, not energy. The factor 1000 converts W/m to kW/m. The exported Q_0 was used directly where available. To facilitate dimensionless comparison between different configurations, the phase-change time was normalized with respect to the finless insulated reference case T25. The normalized melting and solidification times are defined as.

$$\Pi_m(X) = t_m(MVF = X) / t_{m,T25}(MVF = X) \quad (17)$$

$$\Pi_s(X) = t_s(MVF = X) / t_{s,T25}(MVF = X) \quad (18)$$

where t_m and t_s are the melting and solidification times, respectively. For melting, $X = 0.7, 0.8$, and 0.9 , while for solidification, $X = 0.3, 0.2$, and 0.1 . Values of $\Pi < 1$ indicate faster phase transition than the finless insulated baseline, while $\Pi > 1$ indicates slower performance. The corresponding normalized time-saving percentage is calculated as.

$$\eta_t = (1 - \Pi) \times 100\% \quad (19)$$

This metric allows the improvement of each configuration to be interpreted independently of the absolute phase-change time.

3. Numerical strategy and model validation

This section outlines the adopted numerical methodology, including mesh sensitivity analysis and the procedures implemented to ensure the reliability and accuracy of the simulation model.

3.1. Finite element method implementation

The governing equations, along with their initial and boundary conditions, were resolved using the finite element method (FEM). This numerical approach was selected due to its robustness in handling the complex source and sink terms that arise from the phase change processes of melting and solidification, as discussed in references [32, 33].

Rather than employing the strong form, the equations were

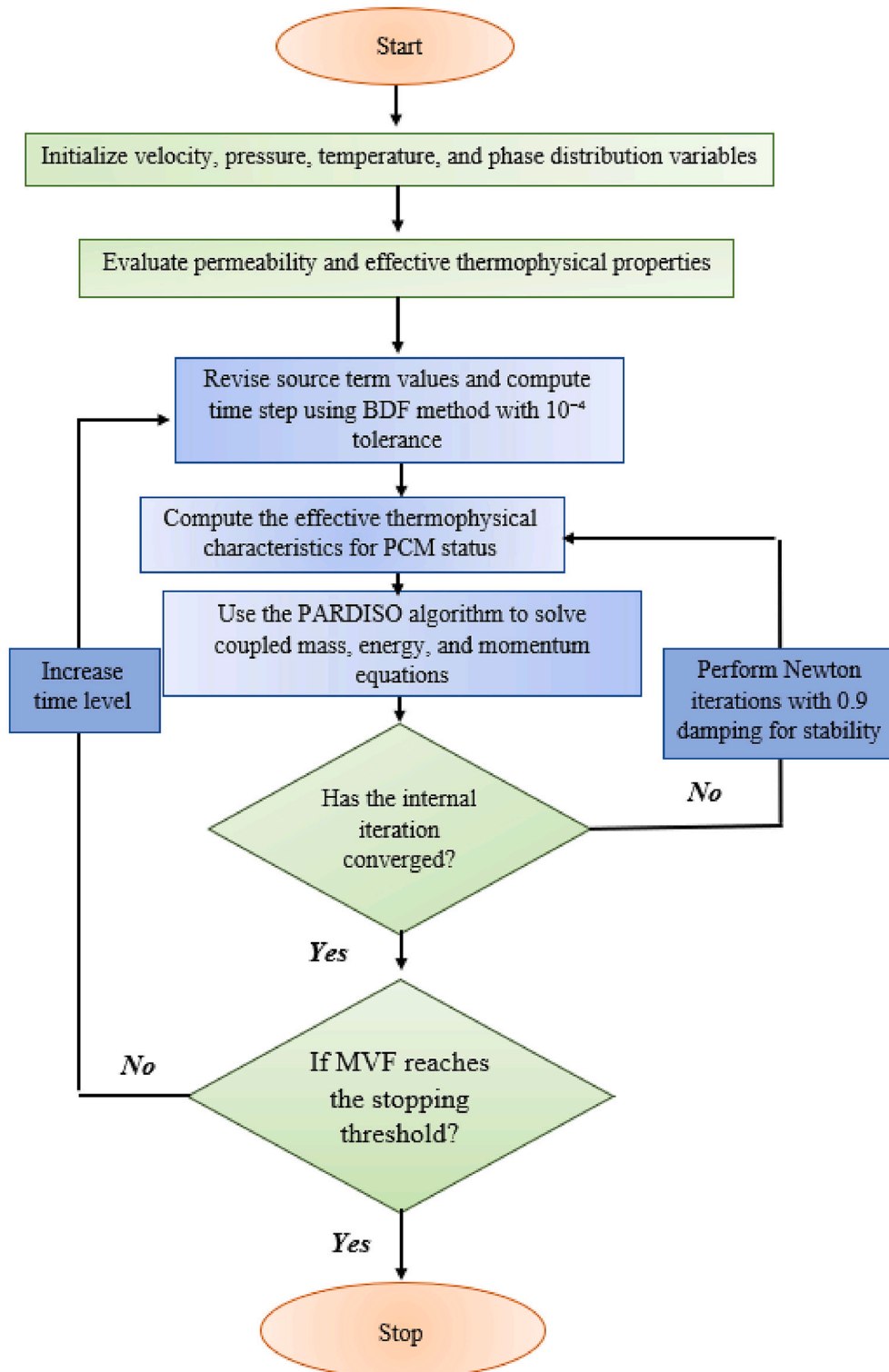


Fig. 5. The steps of numerical approach for simulation of phase transition heat transfer.

reformulated in their weak form to facilitate numerical treatment. The energy and momentum equations were discretized into second-order algebraic systems. Gauss quadrature integration was applied at the element level, resulting in a set of residual algebraic equations. These were iteratively solved using a fully coupled Newton approach [34,35], stabilized using a damping factor set to 0.9 to enhance convergence behavior. To accommodate the computational load, the PARDISO solver, capable of parallel processing on multicore systems, was used to accelerate calculations [36–38].

The simulation time step and update rates were adjusted dynamically throughout the process, maintaining convergence within a relative tolerance of 10^{-4} between successive solution updates. Time integration was performed using a backward difference scheme, with either first- or second-order accuracy, selected based on the numerical stability requirements at each stage.

The FEM framework was particularly suitable for this study due to its capacity to yield consistent and accurate solutions across geometrically partitioned domains. Simulations were initiated from predefined initial conditions and analyzed both charging and discharging cycles of the LHTES system, including phase transition dynamics. The numerical model simultaneously solved the coupled heat transfer and fluid flow equations, incorporating the (f) term to track phase transformation progress.

The computations were terminated once the phase change process was nearly complete. In the case of melting, the end condition was set as a *MVF* of 0.99 or greater. For solidification scenarios, the simulation concluded when *MVF* dropped below 0.01. The overall algorithmic structure of the computational procedure is depicted in Fig. 5.

3.2. Mesh sensitivity analysis

To evaluate the effect of mesh resolution on simulation accuracy, a mesh sensitivity analysis was performed for the reference case T3, considering both melting and solidification. The computational domain was discretized using a non-uniform triangular mesh. The mesh density was varied to generate meshes with different resolutions, denoted by the mesh number *NM*. The details of the mesh configurations, including the number of elements in the PCM domain, copper walls and HTF-side fins, PCM-side fins, and the total element count, are listed in Table 2. The mesh was also aligned along the periodic boundaries to improve numerical accuracy and convergence stability.

To determine the influence of mesh granularity, simulations were conducted at various mesh resolutions, and the results were compared by tracking the *MVF* over time. As shown in Fig. 6, *MVF* curves for different mesh sizes exhibit strong agreement, with higher resolutions leading to convergence of results. Notably, the curves corresponding to *NM* values of 5 and above are nearly indistinguishable, confirming that further refinement beyond this point yields negligible improvement in accuracy. The quantitative deviation of each mesh resolution from the finest mesh is evaluated below to justify the final mesh selection.

Fig. 7 illustrates the mesh layout for *NM* = 6, highlighting key

Table 2

Mesh statistics and *MVF* convergence for various mesh resolutions (*NM*) at $t = 250$ min for the reference case T3.

<i>NM</i>	PCM	Wall and HTF fins	PCM Fins	Total	Solidification <i>MVF</i> at 250 min	Melting <i>MVF</i> at 250 min
2	7658	846	838	9342	0.1429	0.9311
3	16,790	1642	1680	20,112	0.1399	0.9241
4	29,548	2564	2612	34,724	0.1420	0.9169
5	45,868	3878	3768	53,514	0.1412	0.9173
6	65,928	4742	4866	75,536	0.1411	0.9174
7	89,600	6396	6092	102,088	0.1418	0.9177
8	116,892	7544	7432	131,868	0.1417	0.9178

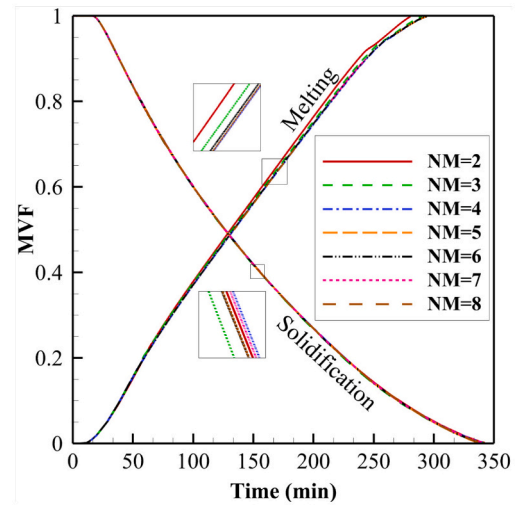


Fig. 6. *MVF* over time during melting and solidification phases for different mesh resolutions (*NM* = 2 to 8).

regions including the tips and edges of the wedge-shaped fins and the outer domain boundaries. The mesh is refined in these areas to accurately capture geometric complexities and gradients in temperature and velocity fields. The phase transition region, which shifts dynamically during charging and discharging, is also covered with a sufficiently fine mesh to maintain resolution accuracy throughout the simulation. Moreover, boundary regions are carefully meshed to resolve steep gradients and capture boundary-layer behavior in thermal and fluid fields.

To quantify the numerical uncertainty associated with mesh resolution and liquid-fraction interpolation, the *MVF* values at $t = 250$ min were compared for both melting and solidification. The finest mesh, *NM* = 8, was used as the reference solution, and the relative *MVF* deviation was calculated as $\varepsilon_{MVF} = |(MVF_{NM} - MVF_{NM=8}) / MVF_{NM=8}| \times 100$. For the selected mesh, *NM* = 6, the deviation from the finest mesh is only 0.42% for solidification and 0.04% for melting. Moreover, for *NM* $\geq$ 5, the maximum deviation remains below 0.43% for solidification and 0.06% for melting. These small deviations indicate that mesh-resolution-related differences in *MVF* at $t = 250$ min are small relative to the performance differences among the studied cases; they do not quantify property, boundary-condition, or model-form uncertainty. Therefore, *NM* = 6, with 75,536 total elements, was selected as the final mesh because it provides a reliable balance between computational accuracy and cost.

3.3. Model validation

To assess the accuracy and reliability of the present numerical framework, the simulation results were compared against previously published experimental and numerical findings from the literature [39,40]. Specifically, the study conducted by [39,40] serves as the benchmark for this comparison. These researches examined the melting characteristics of lauric acid enclosed in a rectangular container, using both experimental observation [39] and computational modeling [40]. The cavity had dimensions of 5 cm in width and 12 cm in height, with the right boundary maintained at a constant temperature of 70 °C, while all remaining walls were thermally insulated to eliminate heat losses.

Fig. 8 presents the *MVF* over time obtained from the current simulation, alongside the data reported by [39,40]. The comparison shows a strong agreement between the trends of the current results and those reported in both experimental and numerical analyses. This consistency supports the credibility and robustness of the developed numerical model in predicting phase change processes under similar boundary and thermal conditions.

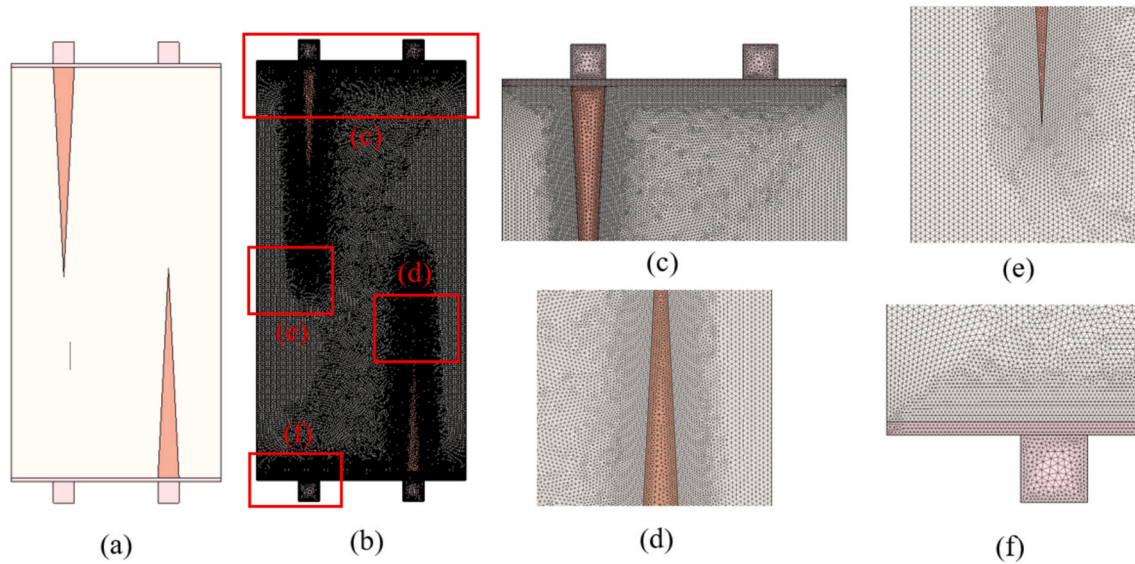


Fig. 7. Mesh configuration corresponding to $NM = 6$. (a) Geometric model; (b) global mesh distribution; (c–f) magnified views of critical regions including wedge-fin tips, wedge-fin edges, and boundaries.

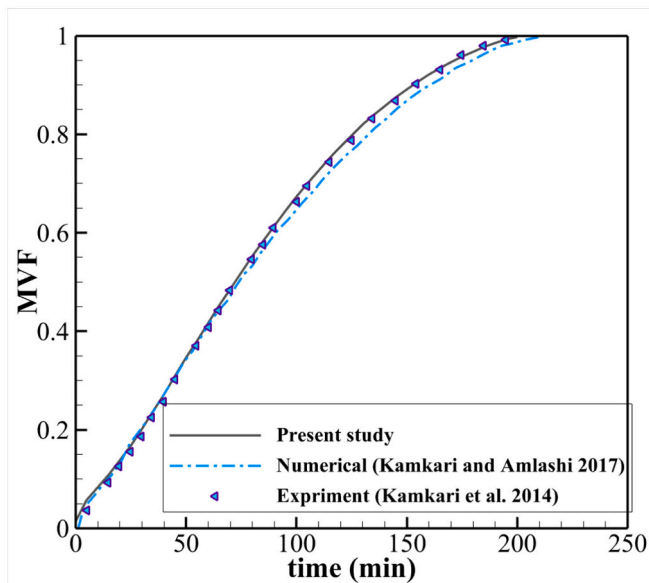


Fig. 8. MVF of this study and that measured by Kamkari et al. [39] and also numerical study of and Kamkari and Amlashi [40].

4. Results and discussions

To investigate the thermal response and phase transition behavior of the LHTES unit, a comprehensive parametric study was conducted using the simulation cases T1–T26. The primary focus was on the effect of fin distribution (top, bottom, both), fin aspect ratio (NR_f), tooth geometry (NR_t), and boundary conditions (periodic vs insulated) on the melting and solidification times and overall thermal performance. A total of 26 simulation cases, including repeated geometries under different operating or boundary conditions, were evaluated as detailed in Table 3. These included variations in heat transfer coefficients (h_{conv}), fin placements, and channel symmetries. For T1–T24, the constant-area relations in Section 2.1 preserve copper mass per unit depth and, within the fixed envelope, PCM volume per unit depth. T25 and T26 intentionally omit PCM-side fins; they therefore contain less copper and more PCM and are used only as unmatched finless baselines.

Table 3

Design specifications and boundary conditions for simulation cases T1–T26.

Case	NR_f	NR_t	h_{conv} (W/m ² .K)	Fin Position	Fin B.C.
T1	0.1	1.0	50	top-bottom	P
T2	0.3	1.0	50	top-bottom	P
T3	0.5	1.0	50	top-bottom	P
T4	0.7	1.0	50	top-bottom	P
T5	0.9	1.0	50	top-bottom	P
T6	0.5	1.0	25	top-bottom	P
T7	0.5	1.0	75	top-bottom	P
T8	0.5	1.0	150	top-bottom	P
T9	0.5	0.75	50	top-bottom	P
T10	0.5	1.25	50	top-bottom	P
T11	0.5	1.50	50	top-bottom	P
T12	0.5	0.75	50	both bottom	P
T13	0.5	1.0	50	both bottom	P
T14	0.5	1.25	50	both bottom	P
T15	0.5	1.5	50	both bottom	P
T16	0.5	0.75	50	both top	P
T17	0.5	1.0	50	both top	P
T18	0.5	1.25	50	both top	P
T19	0.5	1.5	50	both top	P
T20	0.1	1.0	50	top-bottom	I
T21	0.3	1.0	50	top-bottom	I
T22	0.5	1.0	50	top-bottom	I
T23	0.7	1.0	50	top-bottom	I
T24	0.9	1.0	50	top-bottom	I
T25	–	–	–	–	I
T26	–	–	–	–	P

Note: The dashes for T25 and T26 denote finless baseline cases. Because the PCM-side fins are removed, these cases do not satisfy the constant-copper/constant-PCM-volume condition applied to T1–T24.

This work is a comparative parametric assessment, not a formal optimization. No objective function, constraints, or search algorithm were applied, and each parameter effect is interpreted through matched comparisons. In particular, T8 retains the T3 geometry and differs chiefly in its prescribed convective heat transfer coefficient (150 rather than 50 W/m².K); its shortest phase-change times therefore identify the best-performing evaluated operating case, not a jointly optimized geometry.

Fig. 9 visualizes the geometric configurations for all studied cases (T1–T26). Each sub-figure represents a distinct LHTES module with different fin distributions and channel aspect ratios, providing a clear

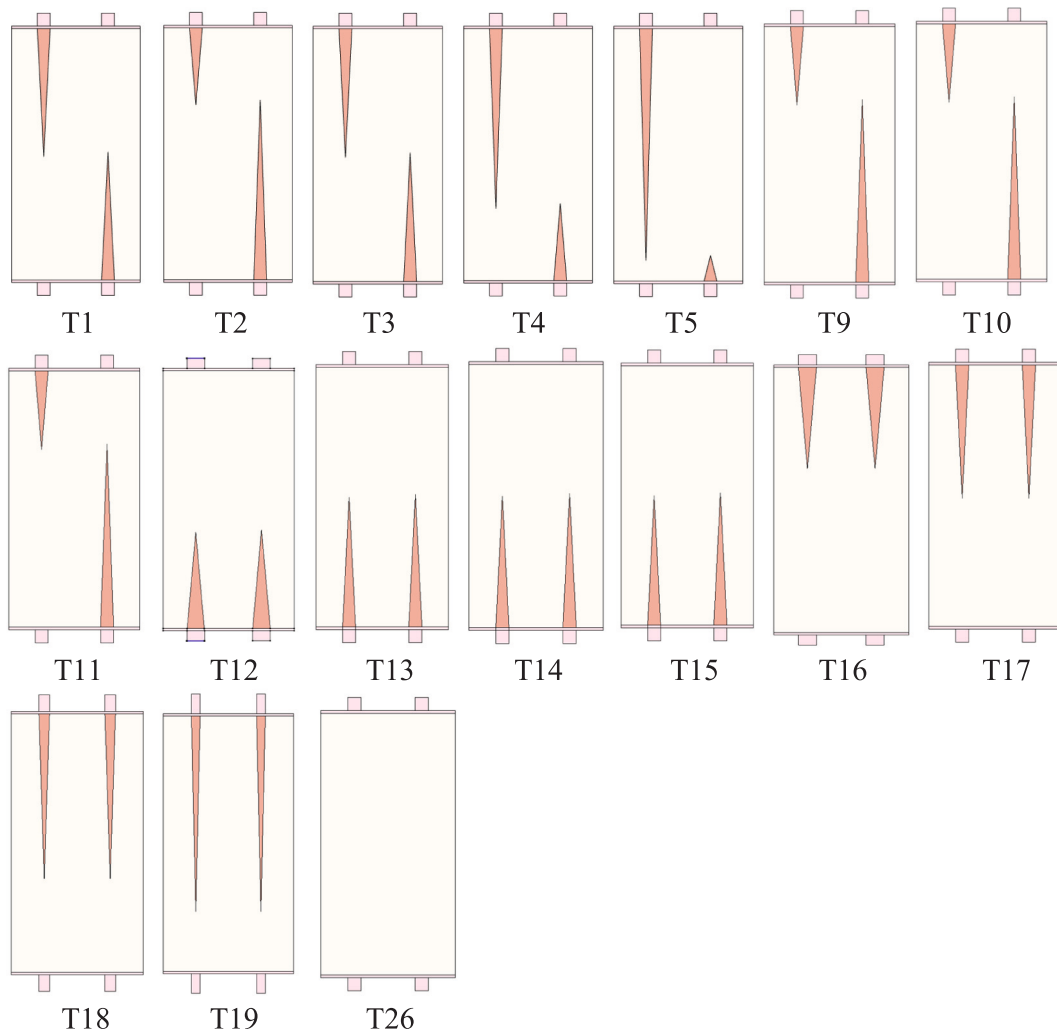


Fig. 9. Geometric configurations of the LHTES unit for simulation cases T1 to T26.

view of the wedge-shaped fin arrangements that influence the heat transfer pathways.

Table 3 outlines the governing design parameters for each configuration, including fin placement (top, bottom, or both), fin boundary condition (periodic 'P' or insulated 'I'), and key non-dimensional shape factors such as NR_f (fin height ratio) and NR_t (tooth shape ratio). These geometric parameters were carefully chosen to enable a comparative assessment of each variable's role in PCM melting/solidification dynamics.

Table 4 presents the corresponding thermal response metrics for each case, reporting the time required to reach specific melt volume fractions ($MVF = 0.7, 0.8$, and 0.9) during melting, and $MVF = 0.3, 0.2$, and 0.1 during solidification. These benchmarks allow for a quantitative comparison of the system's charging and discharging efficiency.

Examining fin height ratios, significant differences emerge. Cases T1–T5 represent variations of NR_f from 0.1 to 0.9 under periodic boundary conditions with top-bottom fin placement. Since the total fin height and copper amount are kept constant, changing NR_f redistributes the available fin length between the top and bottom PCM regions rather than adding extra conductive material. Case T3, with $NR_f = 0.5$, achieves the shortest melting times in this subgroup, reaching $MVF = 0.7, 0.8$, and 0.9 in 187, 215, and 244 min, respectively. This best-performing tested ratio reflects a balance between two competing effects. When NR_f is too small or too large, the fin length becomes biased toward one side of the PCM domain, leaving the opposite side to be

heated or cooled mainly through a longer PCM conduction path. Conversely, a balanced fin-height ratio provides conductive access to both sides of the domain, reduces the maximum PCM-to-fin distance, and improves the uniformity of the phase front. Therefore, the moderate value $NR_f = 0.5$ minimizes the combined vertical and lateral thermal resistance more effectively than the extreme ratios. The extreme cases T1 and T5 showed melting durations around 8% longer than the best-performing T3 case, confirming the penalty associated with unbalanced fin distribution.

Comparing boundary conditions between periodic and insulated (T20–T24) highlights another important performance factor. Insulated conditions generally increased melting and solidification times. T22 with insulated boundaries required 190 min to reach $MVF = 0.7$, slightly higher than the periodic counterpart T3 (187 min), translating to a modest yet clear 1.6% increase in duration. More pronounced differences occur at higher NR_f values; for instance, T24 ($NR_f = 0.9$, insulated) displayed an extended melting duration of 210 min ($MVF = 0.7$), representing a 5% increase over T5 (periodic).

The convective heat transfer coefficient (h_{conv}) was another significant variable investigated through cases T6–T8 (25 W/m².k to 150 W/m².k). Higher convection markedly enhanced thermal performance. Notably, T8 with the highest h_{conv} (150 W/m².k) significantly reduced melting times to 103 min for $MVF = 0.7$, an impressive 44.9% reduction compared to T3 (50 W/m².k). Conversely, reduced h_{conv} in T6 significantly impaired thermal efficiency, nearly doubling the melting time

Table 4Phase transition times for each simulation case based on MVF thresholds.

Case	Melting time (min)			Solidification time (min)		
	$MVF = 0.7$	$MVF = 0.8$	$MVF = 0.9$	$MVF = 0.3$	$MVF = 0.2$	$MVF = 0.1$
T1	202	234	269	202	249	307
T2	189	223	259	190	227	281
T3	187	215	244	189	226	270
T4	189	217	249	190	227	282
T5	200	232	267	202	249	308
T6	299	344	392	302	353	411
T7	147	168	191	150	183	223
T8	103	118	135	112	140	176
T9	231	264	298	251	309	380
T10	162	187	213	157	185	220
T11	141	163	185	137	160	185
T12	286	331	378	1064	1214	1364
T13	240	272	308	289	366	466
T14	213	241	273	237	292	358
T15	195	224	255	204	251	311
T16	274	325	378	311	411	528
T17	210	254	302	210	284	376
T18	169	202	242	158	197	264
T19	144	169	197	138	161	192
T20	250	293	339	235	266	297
T21	209	251	298	198	237	303
T22	190	218	249	189	227	271
T23	193	221	256	200	242	308
T24	210	242	277	238	291	358
T25	440	507	576	639	804	987
T26	397	454	513	648	813	996

(299 min at $MVF = 0.7$) relative to T3, confirming convection's crucial role in enhancing heat transfer.

Tooth shape ratios (NR_t) are scrutinized in cases T9–T11, revealing nuanced effects on thermal dynamics. Increasing NR_t from 0.75 (T9) to 1.5 (T11) led to a clear trend of decreasing melting times. Specifically, T11 with NR_t of 1.5 achieved $MVF = 0.7$ in only 141 min, a notable 39% improvement over T9 (231 min). Similar trends are observed during solidification; T11 solidified to $MVF = 0.3$ in 137 min, nearly half the time of T9 (251 min), confirming that higher tooth ratios considerably improve heat penetration and phase transition efficiency.

Fin placement had a significant impact, as shown by cases T12–T19. To isolate the effect of fin position, cases with the same tooth ratio should be compared. For $NR_t = 1.0$, the top-fin case T17 reached $MVF = 0.7$ in 210 min, whereas the corresponding bottom-fin case T13 required 240 min. For $NR_t = 1.5$, T19 required only 144 min, compared with 195 min for T15. Thus, top-fin placement reduced the melting time by about 12.5–26.2% compared with bottom-fin placement for these representative cases. A similar trend was observed during solidification: T17 reached $MVF = 0.3$ in 210 min compared with 289 min for T13, while T19 required 138 min compared with 204 min for T15. These results indicate that the advantage of top-fin placement is not only a geometric effect but also results from its interaction with thermal stratification and buoyancy-driven heat redistribution.

The normalized phase-change times, provided in Table 5, provide a clearer comparison of the relative importance of each design modification. The finless periodic case T26 gives only a modest improvement in melting compared with T25 and shows almost no improvement during solidification, indicating that periodicity alone cannot compensate for the absence of PCM-side fins. In contrast, the reference finned case T3 reduces Π_m to 0.43 and Π_s to 0.30, confirming that the main improvement is produced by adding conductive fin pathways. The best-performing case T8 further reduces Π_m and Π_s to 0.23 and 0.18, respectively, because the higher convective coefficient decreases the HTF-side thermal resistance. The comparison between T15 and T19 also shows that top-fin placement provides lower normalized melting and solidification times than bottom-fin placement at the same tooth ratio. Therefore, the normalized metric confirms that both the expected effect

Table 5

Normalized phase-change time for representative configurations using the finless insulated case T25 as the reference.

Case	Main feature	Π_m at $MVF = 0.7$	Melting time saving (%)	Π_s at $MVF = 0.3$	Solidification time saving (%)
T25	Finless, insulated baseline	1.00	0.0	1.00	0.0
T26	Finless, periodic	0.90	9.8	1.01	−1.4
T3	Reference finned case	0.43	57.5	0.30	70.4
T8	High h_{conv} , same geometry as T3	0.23	76.6	0.18	82.5
T11	High tooth ratio, top-bottom fins	0.32	68.0	0.21	78.6
T15	Bottom fins, $NR_t = 1.5$	0.44	55.7	0.32	68.1
T19	Top fins, $NR_t = 1.5$	0.33	67.3	0.22	78.4

of higher h_{conv} and the geometry-specific effects of fin ratio, tooth ratio, and fin placement contribute to the overall enhancement.

Extreme scenarios without fins (T25 and T26) provided critical baseline comparisons. Both cases highlighted significantly prolonged phase transitions, underscoring fins' pivotal role in enhancing heat transfer. T25 (insulated boundary without fins) required an extensive 440 min to reach $MVF = 0.7$ during melting, whereas the best-performing T8 case required only 103 min. Thus, T25 required approximately 327% longer melting time than T8. Similarly, T26 (periodic boundary without fins) required 397 min, corresponding to approximately 285% longer melting time than T8. Equivalently, T8 reduced the melting time by approximately 76.6% and 74.1% relative to T25 and T26, respectively. These results emphasize the substantial heat-transfer enhancement achieved by the evaluated fin geometry and operating conditions.

Quantifying performance differences, the highest-convection case T8 achieved the shortest melting time (103 min at $MVF = 0.7$), significantly outperforming baseline finless scenarios (T25 and T26) by approximately 76%. This dramatic reduction underscores convection's critical role in achieving rapid energy storage.

Considering the cumulative data and thermal contours provided in Fig. 9, clear design-selection trends emerge from this comparative parametric study. Moderate fin height ($NR_f = 0.5$), increased tooth ratio ($NR_t = 1.5$), higher convective heat transfer coefficients ($150 \text{ W/m}^2\text{K}$), and strategic fin placement (top and bottom periodic) consistently delivered superior performance. Conversely, low convection rates, reduced tooth ratios, extreme fin heights, and insulated boundaries notably impaired efficiency.

Ultimately, this comparative analysis underscores that targeted adjustments in fin geometry, orientation, and boundary conditions can significantly enhance phase transition efficiency in LHTEs applications. Using the best-performing tested configurations, such as T8 and T11, could lead to substantial improvements in thermal energy storage systems, enhancing practical feasibility and operational performance across diverse thermal management applications.

During melting, the configurations showed that moderate geometric parameters, such as a fin height ratio (NR_f) of 0.5, provided the best heat-transfer performance among the tested ratios. For instance, case T3 consistently melted faster than scenarios with extreme NR_f values (T1 at 0.1 and T5 at 0.9). However, when considering higher melt fractions ($MVF = 0.9$), the advantages of the best-performing tested geometry slightly diminish. Specifically, while T3 maintained shorter melting times, the performance gap between the best- and lower-performing tested scenarios narrowed. This indicates that as melting nears

completion, the precise geometry of the fins becomes slightly less critical, although the best-performing tested design still provides measurable benefits.

Solidification showed an opposite trend. Differences in heat transfer effectiveness between configurations became more pronounced as solidification approached completion ($MVF = 0.1$). For instance, the best-performing case T8 (high convective heat transfer coefficient of $150 \text{ W/m}^2\cdot\text{k}$) drastically outperformed less efficient cases like T3 at lower solid fractions. While T8 was already effective at higher fractions ($MVF = 0.3$), its advantage expanded as solidification progressed toward completion, underscoring how crucial efficient heat transfer is in the final stages of energy retrieval.

During melting, heat transfer initially occurs by conduction because the PCM is solid. As the liquid region develops, buoyancy-driven convection becomes stronger and helps redistribute heat, making the relative influence of fin geometry less critical near the later stages of melting. In contrast, solidification starts with the PCM mainly in the liquid state, so natural convection can be important initially. However, as the solid layer grows, the remaining liquid region becomes confined and convection is gradually suppressed; therefore, fins become more important during the middle and later stages of solidification by providing conductive heat-transfer pathways between the PCM and the cold walls. This interpretation is consistent with the experimental results of Kamkari and Shokouhmand [14], who reported that the melting enhancement ratio of partial fins decreases with time after reaching a maximum, and Abdi et al. [41], who showed that fins have a stronger enhancing effect during solidification than melting, with mean-power improvements of 395% and 90%, respectively.

Moreover, cases without fins (T25 and T26) emphasized the importance of fin presence and orientation. These finless scenarios had significantly longer melting and solidification times, highlighting fins' vital role in enhancing heat transfer within LHTES systems. Specifically, solidification processes in finless cases saw extreme increases in duration at lower solid fractions, indicating especially poor heat transfer efficiency.

The comparison of periodic and insulated boundary conditions further demonstrated clear differences. Insulated boundaries generally increased melting and solidification times across all MVF thresholds. However, the differences became slightly less pronounced at higher MVF thresholds during melting and significantly more pronounced at lower MVF thresholds during solidification, emphasizing that effective boundary management can significantly enhance thermal performance,

particularly during energy retrieval phases.

The contours plotted are based on the temperature difference variable (ΔT), capturing the absolute change in temperature with respect to the initial condition. Given that the initial and operating temperatures are defined relative to the PCM fusion temperature T_f ($\pm 15^\circ\text{C}$), the range of ΔT spans from 0°C to 30°C . This visualization approach ensures that thermal gradients and fin-induced heat penetration are clearly observed in the post-processed contour plots.

The combined analysis of Fig. 9, Table 3, and Table 4 forms the basis for discussing the thermal performance of each LHTES configuration. Key observations regarding fin orientation (e.g., T1 vs T5), tooth geometry (e.g., T9 vs T11), and boundary conditions (e.g., T3 vs T25) will be addressed in the following subsections, where melt contours and performance metrics are directly compared to identify the most effective configurations for rapid energy storage and retrieval.

4.1. Fin height ratio (NR_f)

Fig. 10 presents the time evolution of MVF during both melting (Fig. 10 (a)) and solidification (Fig. 10 (b)) for different fin height ratios, $NR_f = 0.1$ to $NR_f = 0.9$, corresponding to simulation cases T1 to T5. These configurations maintain a constant fin volume while altering the vertical distribution of copper fins.

During the melting phase (Fig. 10 (a)), all curves show closely overlapping trends in the early and mid-stages of the process (up to approximately 200 min). However, as the MVF approaches 0.9, the curves begin to diverge more significantly. This divergence indicates that fin distribution still affects the location and morphology of the remaining solid PCM during the later part of melting. However, at this stage, a substantial liquid region has already formed; therefore, buoyancy-driven convection is present and contributes to heat redistribution. The fin-height effect should therefore be interpreted mainly as the ability of the fins to conduct heat into the remaining solid and weak-circulation regions, rather than as evidence that the whole melting process is purely conduction-dominated. The steeper slopes and early attainment of full melting observed in T3 and T4 (with $NR_f = 0.5$ and 0.7) suggest that these configurations achieve better vertical thermal penetration and more uniform melting, particularly in the lower PCM zones.

The best-performing tested NR_f can be understood as a competition between conductive penetration and heat-spreading resistance. For very small NR_f , most of the fin length is assigned to the bottom side, while the

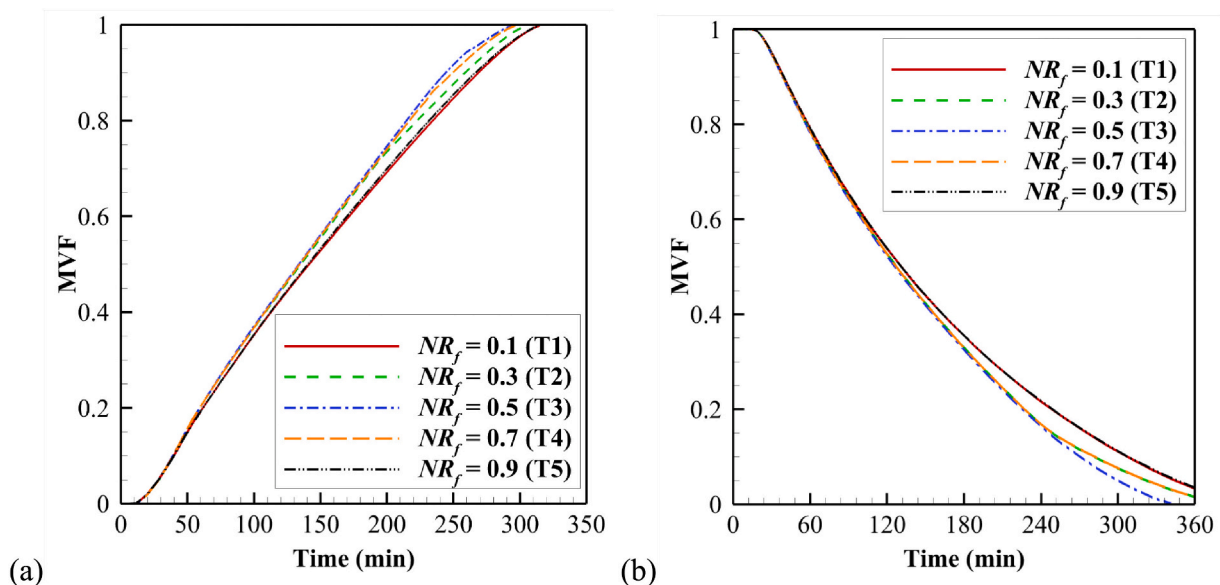


Fig. 10. Variation of MVF during (a) melting and (b) solidification.

top fin becomes too short to reduce the thermal resistance in the upper PCM region. For very large NR_f , the opposite occurs: the top region is well connected to the copper pathway, but the lower and central PCM regions remain farther from the fin tips and depend more strongly on

conduction through low-conductivity PCM. In addition, excessively long and slender wedge fins tend to create localized vertical thermal paths but weaker lateral heat spreading, which can leave unmelted or unso-
lidified pockets between adjacent fin-influenced regions. In contrast, the

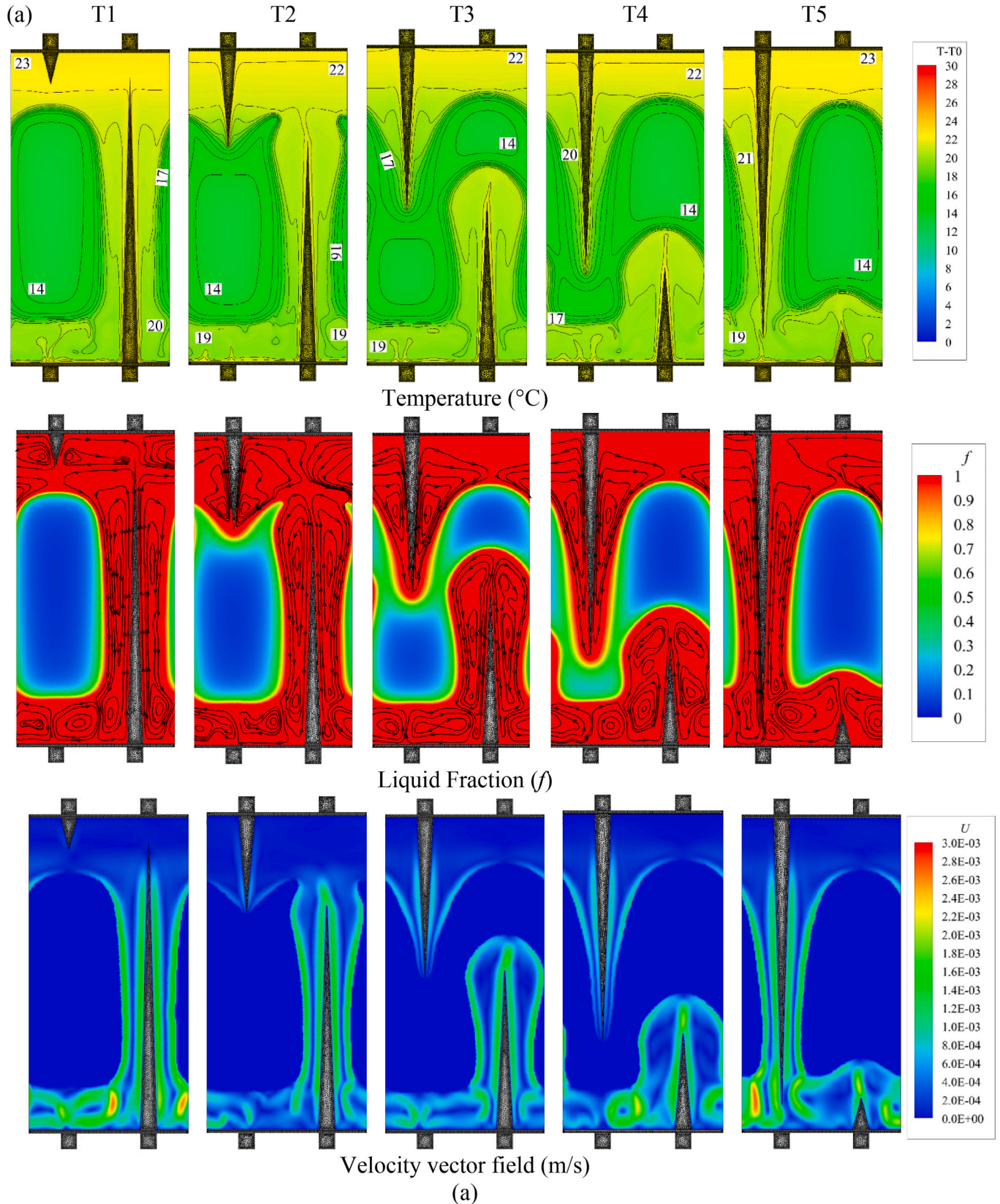


Fig. 11. Temperature, liquid fraction, and velocity field contours for T1–T5 at $t = 200$ min during (a) melting and (b) solidification.

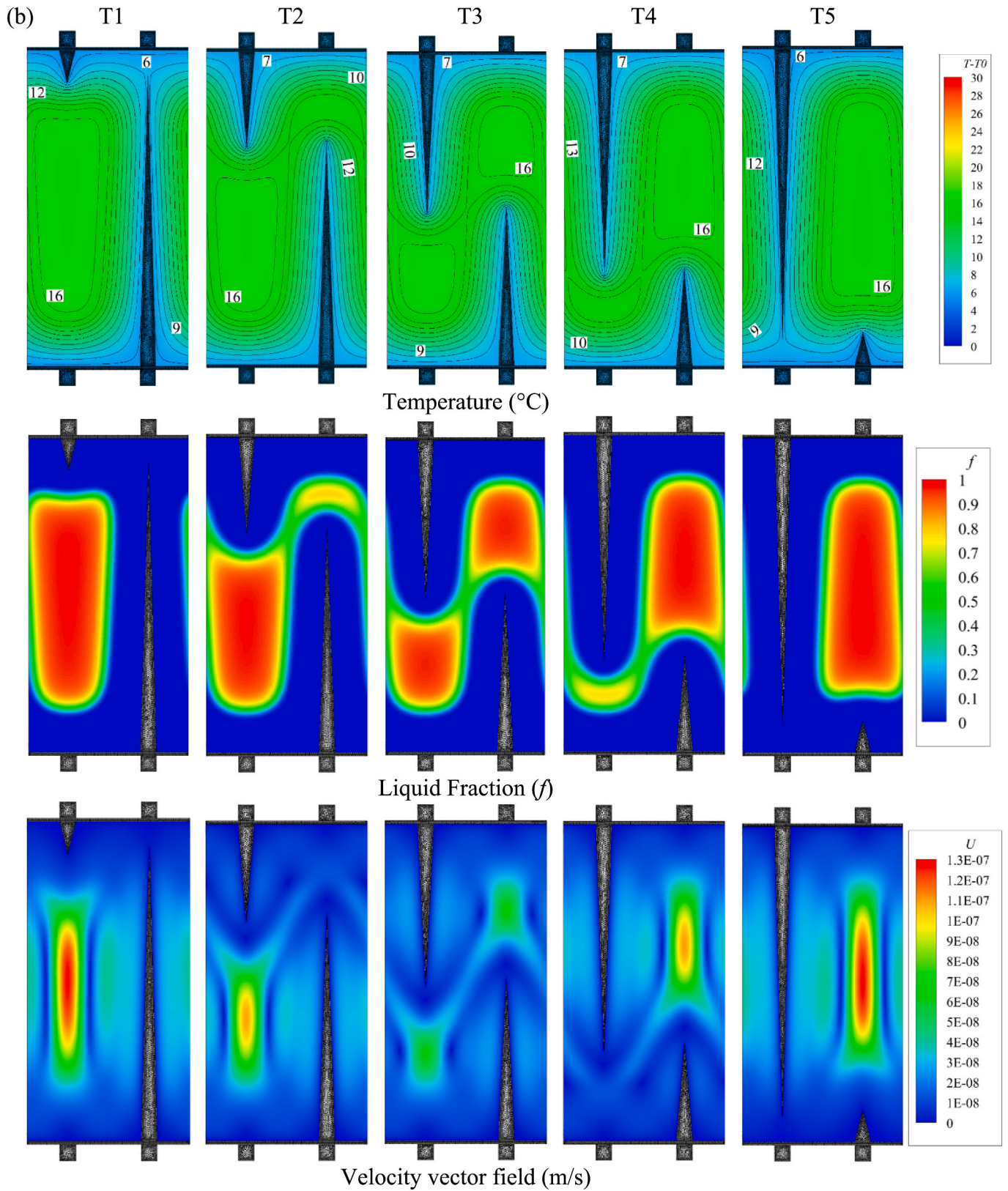


Fig. 11. (continued).

balanced configuration $NR_f = 0.5$ provides comparable top and bottom conductive pathways, reduces the maximum distance between PCM and copper surfaces, and avoids excessive localization of heat transfer near only one side of the domain. Thus, the performance of this fin height ratio results from minimizing the combined PCM conduction distance

and fin spreading resistance.

This behavior is supported by the temperature and liquid fraction contours at $t = 200$ min shown in Fig. 11 (a). For T3 and T4, molten zones extend deeper and more uniformly across the domain, particularly near the bottom region where natural convection is weakest. In these

configurations, the fin distribution provides sufficient penetration into both the upper and lower PCM regions while maintaining effective lateral heat spreading. In contrast, T1 and T2 bias the fin length toward one side of the domain, causing the opposite side to rely more heavily on slow PCM conduction. Conversely, T5 shifts the conductive pathway strongly toward the upper side and produces more localized vertical heat-transfer regions, but weaker lateral spreading in the lower and central PCM regions. This imbalance results in isolated unmolten regions and a less uniform phase front.

This behavior manifests physically in the formation of elongated melt fingers around thin fins (T5), while thick fins (T1) produce flatter, more cohesive molten layers. Importantly, fin effects are most influential before the melt front surpasses the fin height. Once the melt front exceeds the fin length, the molten PCM above forms a relatively homogeneous zone where fin-induced conduction becomes less effective.

Similarly, the solidification curves in Fig. 10 (b) exhibit minimal separation at early times but diverge significantly as the *MVF* decreases below 0.3. Here, the influence of fin distribution becomes critical in expediting heat extraction, especially from regions farthest from the cooling surface. The rapid decline in *MVF* for T3 and T4 again highlights the benefit of balanced or bottom-biased fin placement in improving solidification uniformity and efficiency.

To elucidate these late-stage differences, contour plots of temperature, liquid fraction, and velocity at $t = 200$ min are provided in Fig. 11 (a) (melting) and Fig. 11 (b) (solidification). These snapshots are taken at a critical moment when the curves in Fig. 10 begin to diverge. For melting, Fig. 11(a) corresponds to a stage where a substantial liquid region has already formed; therefore, buoyancy-driven convection is present and is not ignored. However, the differences among the fin-height-ratio cases are mainly associated with fin-guided thermal penetration into the remaining solid and weak-circulation regions. For solidification, Fig. 11(b) corresponds to a stage where the velocity magnitude is much lower, and heat transfer is predominantly controlled by conduction through the growing solid layer and the fins.

In the temperature fields, the T3 and T4 cases display deeper thermal penetration along the fins, with higher temperatures reaching closer to the bottom PCM boundary. The liquid fraction contours support this observation, showing larger molten regions for T3 and T4, especially in the central and lower zones. In contrast, T1 and T5 exhibit more solid PCM in these regions, correlating with their slower *MVF* growth in the plot.

The velocity fields in Fig. 11(a) confirm that natural convection is present during melting. The velocity magnitude is on the order of 10^{-3} m/s, with local maxima approaching 3×10^{-3} m/s. These velocities are mainly localized near the melt front and fin-influenced liquid regions; therefore, convection contributes to heat redistribution in the molten PCM. Nevertheless, the differences among T1–T5 are still primarily associated with fin-guided thermal penetration into the remaining solid and weak-circulation regions, rather than with a fundamentally different large-scale circulation pattern.

On the other hand, the temperature contours during solidification similarly highlight the enhanced cooling in T3 and T4 configurations, where lower temperatures extend deeper into the PCM. The liquid fraction fields show more advanced solidification (i.e., smaller f values) in these cases, while T1 and T5 retain larger liquid cores. This agrees with the *MVF* plot where T3 and T4 descend faster in the late stage.

These spatial non-uniformities in freezing are tied to the fin layout: short and thick fins (T1) solidify efficiently near the base but fail to penetrate upward, while tall and thin fins (T5) extract heat vertically but leave residual melt in horizontal gaps. Again, T3 and T4 offer a geometric compromise that minimizes both vertical and horizontal thermal resistance. This effect becomes particularly important during solidification because natural convection is strongly suppressed at later times; therefore, the remaining liquid must release heat mainly through the conductive network formed by the fins and the growing solid PCM layer.

In contrast, during solidification, the velocity magnitudes in Fig. 11

(b) remain very low, mostly in the range of 10^{-8} – 10^{-7} m/s, with peak values of approximately 1.3×10^{-7} m/s. This confirms that buoyancy-driven circulation is strongly suppressed at this stage. Therefore, the favorable heat removal performance of balanced or bottom-oriented fin designs mainly stems from improved conductive heat-transfer paths rather than fluid circulation.

4.2. Convective heat transfer coefficient (h_{conv})

Fig. 12 illustrates the impact of varying convective heat transfer coefficients (h_{conv}) on the phase transition behavior of the LHTES unit. Cases T6 to T8, with h_{conv} values ranging from 25 to 150 W/m²·K, are compared during the melting (Fig. 12 (a)) and solidification (Fig. 12 (b)) phases. In both plots, the early and intermediate segments of the curves show relatively modest variation across cases. However, as the processes approach completion (high *MVF* during melting or low *MVF* during solidification), the curves start to diverge significantly. This divergence indicates that the role of convective enhancement becomes increasingly pronounced near the end of the phase change processes.

To better understand the mechanisms underlying this divergence, contour maps of temperature, liquid fraction, and velocity at $t = 140$ min are presented in Fig. 13 (a) (melting) and Fig. 13 (b) (solidification). This specific moment was chosen because it aligns with the onset of major deviations among the *MVF* curves in Fig. 12, before nonlinear convection effects dominate and while conduction is still the primary mode of thermal transport.

The temperature contours reveal deeper thermal penetration into the PCM for cases with higher h_{conv} , particularly T7 and T8. These cases exhibit stronger temperature gradients along the HTF-PCM interface, which directly correlate with increased heat flux into the storage medium. T6, with the lowest $h_{\text{conv}} = 25$ W/m²·K, shows a relatively shallow heat front and reduced penetration into the PCM, consistent with its significantly slower melting rate in Fig. 12 (a).

The liquid fraction fields further validate these findings: case T8 demonstrates a nearly complete melt in the upper and middle PCM regions, whereas T6 retains a substantial solid core. Intermediate cases T3 and T7 show gradual progression between these two extremes. Notably, this trend matches the separation in *MVF* curves in Fig. 12 (a), where T8 reaches full melting approximately 350 min earlier than T6.

The velocity fields during melting remain modest (on the order of 10^{-3} m/s), though some enhanced convective patterns begin to emerge in T7 and T8. These flows are localized and do not dominate the overall heat transfer yet. Thus, the major differences in performance at this stage are still attributed primarily to conduction, modulated by the imposed h_{conv} at the boundaries.

In the solidification contours, similar behavior is observed in reverse. Higher h_{conv} values yield stronger cooling fronts. Case T8 exhibits the coldest and deepest penetration of the temperature field, while T6 retains warmer temperatures near the PCM core. The liquid fraction plots show that T8 has solidified the majority of its volume, whereas T6 still contains a large molten zone.

The velocity fields in the solidification regime are markedly weaker (on the order of 10^{-7} m/s), further reinforcing that conduction is the primary mechanism governing the cooling process at this early stage. Thus, differences in *MVF* decline rates in Fig. 12 (b), particularly the rapid drop seen in T8, can be directly attributed to more efficient conductive heat extraction enabled by the elevated h_{conv} .

This behavior is caused by the reduction of the external convective thermal resistance at the HTF-side boundary. Since T6, T3, T7, and T8 have the same geometry, the differences among these cases are not due to fin shape, but to the imposed convective heat transfer coefficient. A higher h_{conv} increases the heat flux at the channel-wall and tooth-side surfaces according to $q = h_{\text{conv}} (T_{\text{conv}} - T)$. As a result, the copper walls and fins reach higher temperatures during melting and lower temperatures during solidification, producing steeper temperature gradients inside the PCM. This accelerates both the advance of the melt

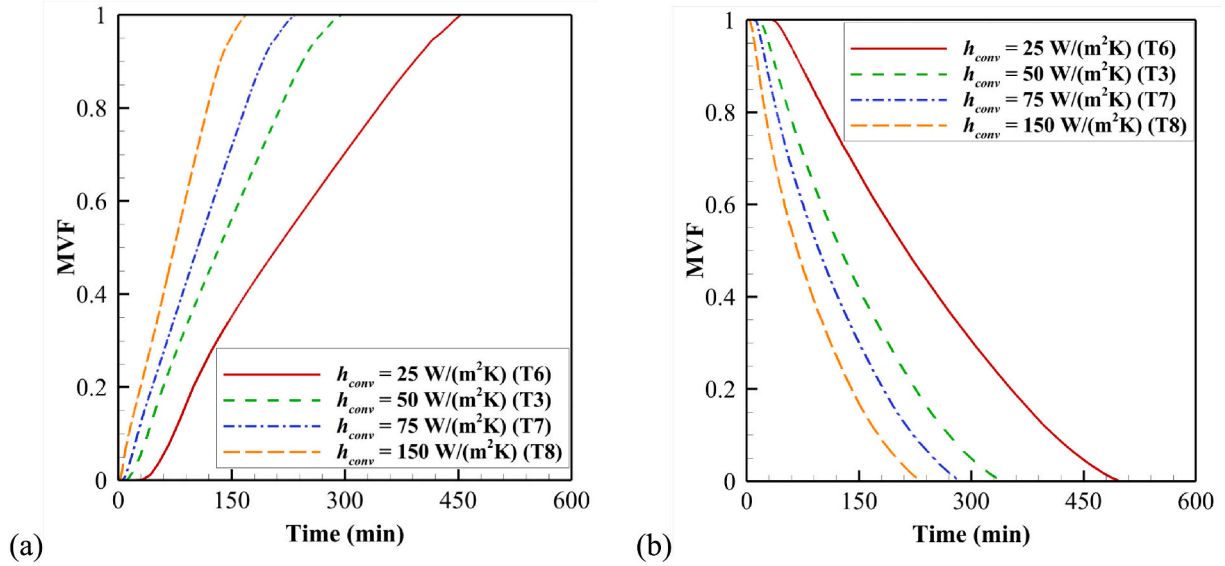


Fig. 12. MVF evolution during (a) melting and (b) solidification for varying convective heat transfer coefficients h_{conv} .

front during charging and the growth of the solid layer during discharging.

During melting, the higher heat input in T7 and T8 also strengthens localized buoyancy-driven circulation in the molten region, as seen in the velocity contours. However, the main difference among these cases remains the boundary heat-transfer rate, which controls the thermal penetration from the HTF side into the PCM through the copper structure. During solidification, the velocity magnitudes are much smaller, so increasing h_{conv} mainly improves conductive heat extraction through the wall, fins, and growing solid layer. Additionally, the enhanced melting and solidification near the bottom regions, particularly in cases with better fin-to-wall contact, can be attributed to the nature of natural convection, which originates from the bottom and rises. Therefore, the fin layout and contact in lower regions are more influential in promoting early phase change dynamics, especially under conduction-dominant conditions.

4.3. Tooth shape ratio (NR_t)

Fig. 14 illustrates the variation in MVF during melting (Fig. 14 (a)) and solidification (Fig. 14 (b)) for four different tooth aspect ratios ($NR_t = 0.75, 1.0, 1.25, 1.50$), corresponding to cases T9, T3, T10, and T11, respectively. This parameter defines the height-to-width ratio of the copper tooth, directly influencing the shape and penetration depth of the heat-conducting fins into the PCM domain.

In both melting and solidification processes, the curves remain closely aligned during the early and intermediate stages. However, as the MVF approaches unity during melting or zero during solidification, significant divergence emerges among the cases. Specifically, case T11 ($NR_t = 1.50$) demonstrates the fastest phase transition, while T9 ($NR_t = 0.75$) lags, showing a much slower progression. To analyze these late-stage variations, temperature, liquid fraction, and velocity contours at $t = 140$ min for melting (Fig. 15 (a)) and $t = 150$ min for solidification (Fig. 15 (b)) are examined. These time frames are strategically chosen to capture the moment just before the MVF curves in Fig. 14 start to significantly separate, under conduction-dominated conditions, with minimal convective influence.

The temperature contours reveal progressively deeper thermal penetration from T9 to T11. In T11, the elongated tooth geometry allows the heat to conduct further into the PCM, establishing a steeper and more uniform temperature gradient, particularly near the base. Conversely, T9 shows more limited thermal diffusion, with cooler zones

persisting deeper in the PCM domain. The liquid fraction fields confirm that T11 has achieved a greater extent of melting at this stage, with a larger central molten region, while T9 retains a thicker layer of solid PCM at the lower interface. T3 and T10 exhibit intermediate behavior, reflecting their aspect ratios and corresponding conductive reach. Velocity contours in this phase remain subdued (on the order of 10^{-7} m/s), confirming that natural convection effects have not yet become dominant, and that the differences in melting rates are primarily attributed to conduction facilitated by geometric variations in the fin structure.

This geometric impact can be further interpreted by observing the distribution of the solid PCM in the liquid fraction contours. In cases with short and thick fins (e.g., T9), the solid zone is relatively shallow vertically but spans more broadly in the lateral direction. In contrast, configurations with slender and elongated teeth (e.g., T11) form tall and narrow solid regions, often leaving melt gaps between adjacent fins due to less lateral heat dispersion. These effects are more pronounced during the melting phase, where slender fins promote localized vertical melting columns, whereas thick fins induce broader but flatter molten zones.

As long as the melting front has not surpassed the fin height, the influence of fin geometry remains significant. Once the melting height exceeds the fin tip, however, a large unified liquid region forms, diminishing the role of individual fins. Additionally, the presence of fins near the bottom region is especially crucial, as natural convection initiates from the lower boundaries and propagates upward. Thus, lower fin penetration ensures early-stage thermal conductivity where it's most effective before buoyancy-driven flow becomes dominant.

A similar trend is observed during solidification. The temperature plots for T11 display lower temperatures throughout the PCM, especially along the extended tooth path, enabling faster heat extraction. T9 shows slower cooling and higher residual temperatures, impeding solidification in deeper regions. The liquid fraction fields again validate the performance differences: T11 and T10 have advanced further in solidification compared to T9. Notably, T11 exhibits a more uniformly distributed solid front, while T9 shows a thick liquid layer near the base, delaying full solidification. The velocity magnitudes during this cooling phase are extremely low (near 10^{-8} m/s), reinforcing the fact that conduction remains the governing mechanism and the geometric modifications are the main contributors to the observed differences in thermal behavior.

Fig. 16 presents the evolution of MVF during melting (Fig. 16 (a)) and solidification (Fig. 16 (b)) for simulation cases T12 to T15, each defined by a different tooth aspect ratio ($NR_t = 0.75$ to 1.50). In all cases,

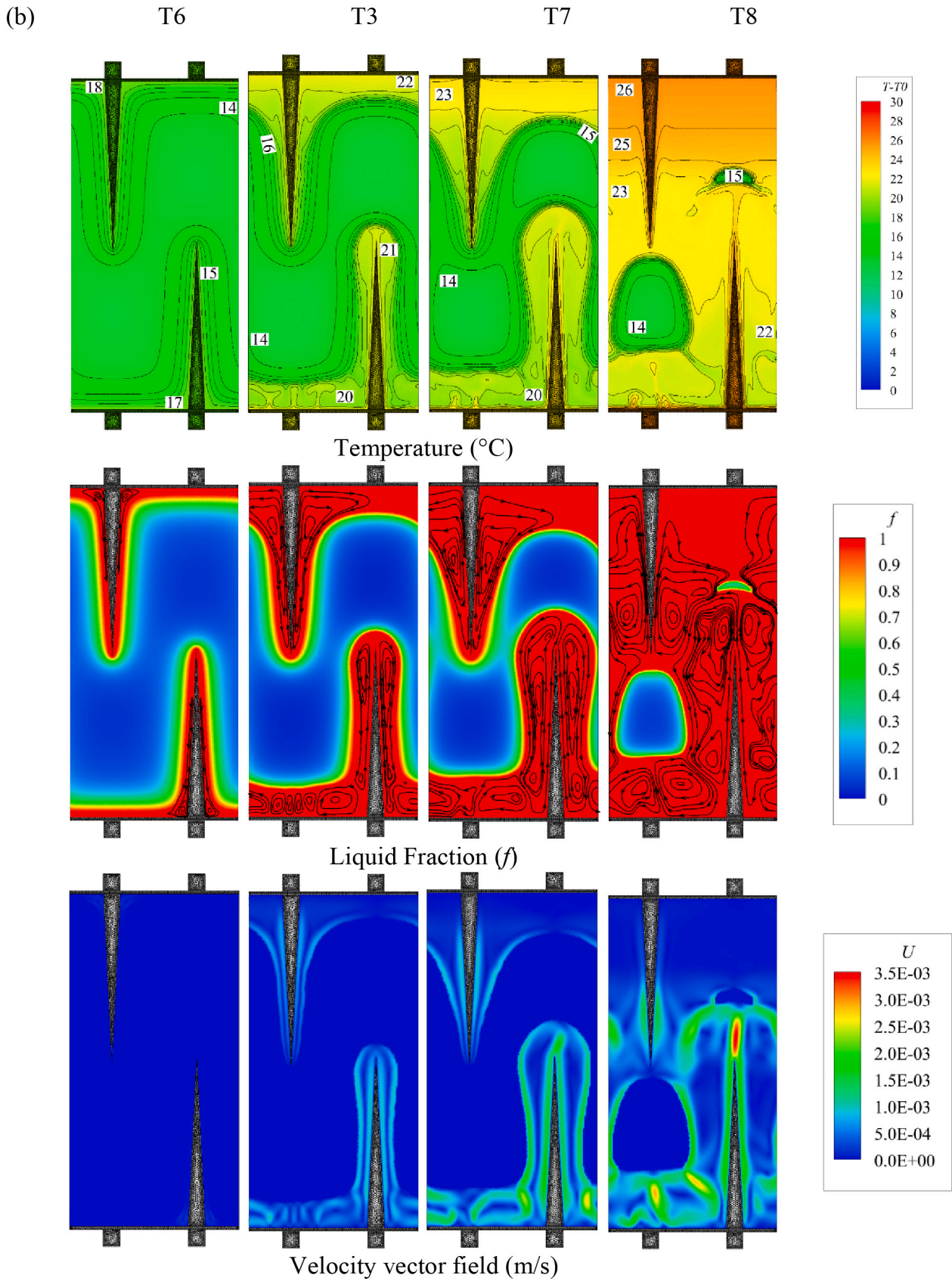


Fig. 13. Temperature, liquid fraction, and velocity contours at $t = 140$ min for cases T6–T8 during (a) melting and (b) solidification.

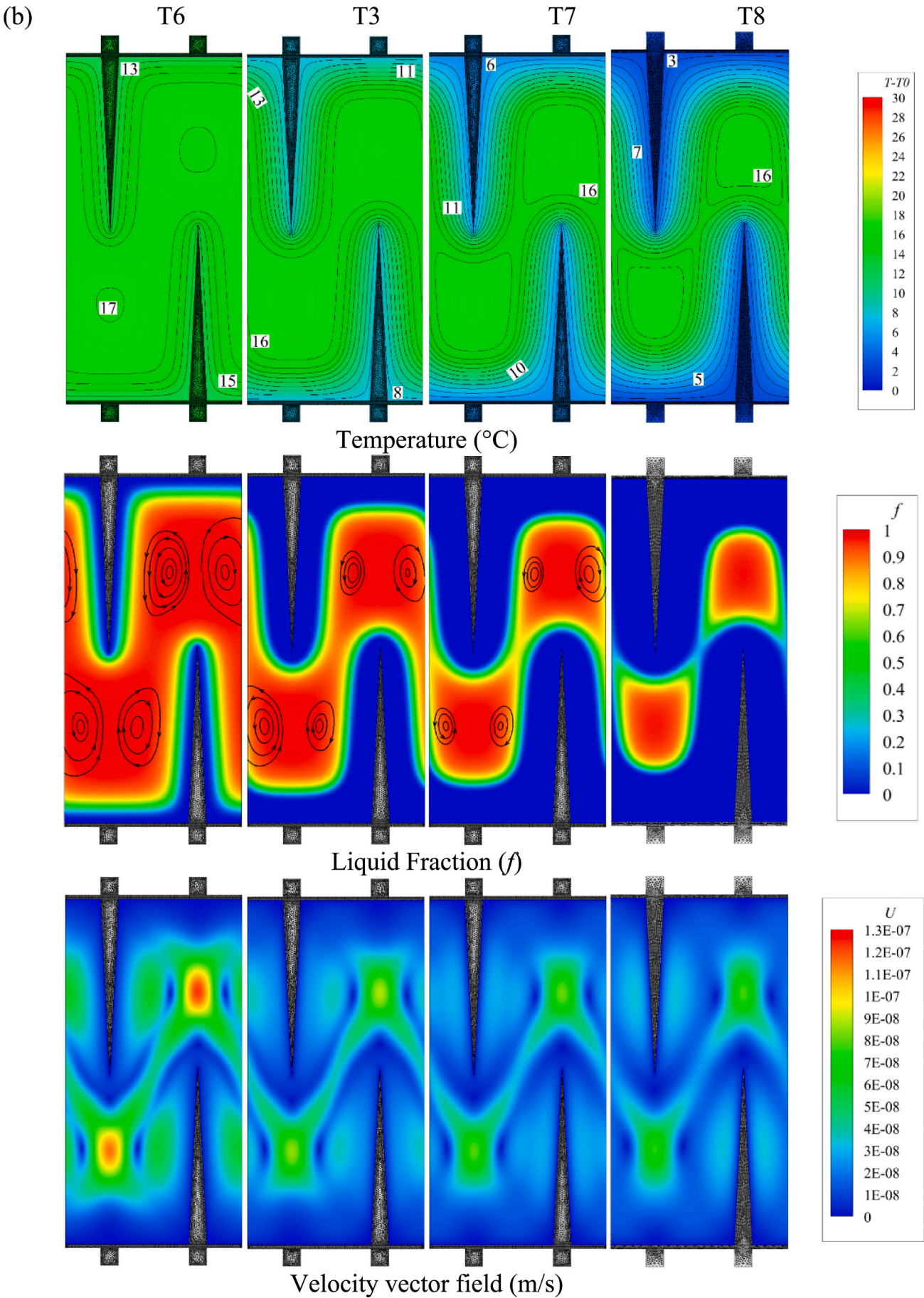


Fig. 13. (continued).

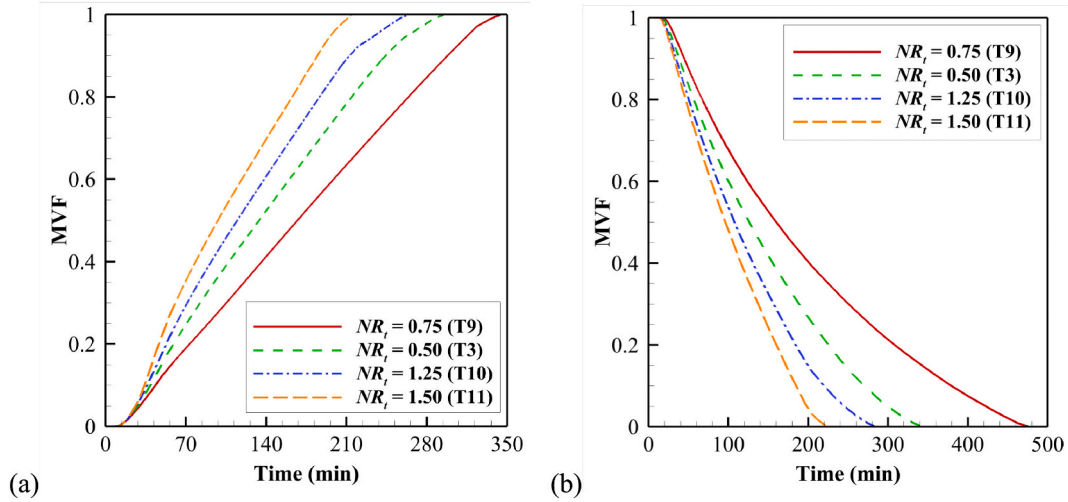


Fig. 14. MVF trends for different tooth aspect ratios NR_t during (a) melting and (b) solidification (cases T9–T11).

the copper fins are placed at the bottom of the PCM domain, emphasizing the role of fin geometry in enhancing thermal diffusion upward into the storage region.

In both plots, the initial portions of the MVF curves are closely aligned, but as the phase change progresses, the trajectories start to diverge noticeably. Particularly during the final third of melting and solidification, the performance gaps widen, with T15 ($NR_t = 1.50$) completing transitions significantly faster than T12 ($NR_t = 0.75$). This divergence reflects the increasing importance of effective thermal conduction paths during the last stages of phase change.

To understand these differences, we examine the temperature, liquid fraction, and velocity vector fields at $t = 170$ min for melting (Fig. 17 (a)) and $t = 180$ min for solidification (Fig. 17 (b)). These times were selected to represent the point at which the MVF curves in Fig. 16 begin to sharply separate. At these times, heat and flow transport are primarily conduction-driven, with convection and nonlinear effects not yet dominant. The temperature contours show that longer fins (higher NR_t) enable deeper and more uniform thermal penetration. T15 demonstrates the most extensive upward diffusion of heat, with isotherms reaching higher into the PCM domain. In contrast, T12 exhibits localized heating near the base with minimal thermal progress upward (Fig. 17 (a)).

The liquid fraction distributions confirm these trends: T15 has significantly melted PCM in its core and upper regions, while T12 retains large solid portions. Cases T13 and T14 show transitional behavior that closely matches their intermediate MVF positions in Fig. 16 (a). The velocity vectors, although present, are still minor (maximum values around 2.4×10^{-3} m/s), indicating that convection is not the primary transport mechanism. The melting improvements in T14 and T15 are due to geometric advantages in conduction, not fluid motion.

A reverse trend is evident in solidification. The temperature fields indicate that T15 has the most efficient cooling profile, with cold zones penetrating deeper toward the base. T12 again lags, with warmer residual PCM and slow thermal withdrawal (Fig. 17 (b)). The liquid fraction maps reveal that T15 has already solidified much of the PCM domain, while T12 retains a significant molten layer in the lower core. T13 and T14 lie between the two, reflecting their expected progression based on tooth geometry.

This behavior can be further explained by analyzing the physical configuration of the fins. In cases with short and thick fins (e.g., T12), the solid zone tends to be wider in the horizontal direction but remains shallow in height. These thick fins facilitate lateral heat conduction but struggle to deliver heat deeply into the upper PCM region. As a result, melting is limited to the lower section and progresses more slowly.

In contrast, taller and thinner fins (e.g., T15) guide heat vertically upward, forming elongated molten zones that rise higher within the

PCM. However, due to the reduced horizontal coverage, small gaps of unmelted PCM may remain between adjacent fins. This vertical stretching of the melt front is especially evident in the liquid fraction fields during melting (Fig. 17(a)), where localized melting zones expand around the tall fins. Once the melt front surpasses the height of the fins, their influence diminishes, and a continuous molten layer forms, with natural convection becoming more dominant. The observed velocities in the present study are at the scale of 10^{-3} m/s which are in the scale of the experimental study reported by Teather and Siddiqui [42].

During solidification, the same geometry causes the solidification front in T15 to develop vertically, while in T12, the front spreads horizontally. Since natural convection assists heat removal from bottom to top, having extended fins located near the base (as in T15) enhances the removal efficiency during early solidification, before conduction dominates the later stages. Velocity vectors during solidification are negligible (on the order of 10^{-8} m/s), confirming that conduction dominates the heat removal process at this stage. There are no signs of large-scale fluid motion that would significantly alter the solidification trends. This is because, during the later stages of solidification, most of the enclosure is in the solid state, with almost zero velocity. The thick solid layer also acts as an insulating layer over the heat transfer surfaces, which reduces the overall heat transfer and the temperature gradients inside the remaining confined liquid regions. Thus, at the later and final stages of solidification, very low velocities are expected. It should be noted that at the initial stage of solidification, the velocity magnitude is on the order of 10^{-3} m/s, which is in very good agreement with the experiments of Teather and Siddiqui [43]. Teather and Siddiqui [43] examined the solidification of Rubitherm RT26 in a domain of about 7 cm and found velocity scales of about 10^{-3} m/s for temperature differences of about 7–15 °C.

Fig. 18 illustrates the MVF evolution during melting (Fig. 18 (a)) and solidification (Fig. 18 (b)) for cases T16 to T19, which all feature fins located at the top of the PCM domain. The only varied parameter here is the tooth aspect ratio NR_t , ranging from 0.75 (T16) to 1.50 (T19).

As seen in the melting plot (Fig. 18 (a)), the MVF curves begin to diverge noticeably after the midpoint of the melting process, with case T19 clearly outperforming others in terms of melting speed. Similarly, in the solidification plot (Fig. 18 (b)), higher NR_t values lead to faster thermal withdrawal and accelerated phase reversal, particularly in the later stages.

However, unlike previous configurations where fins were positioned at the bottom or top and bottom simultaneously, the top-only placement constrains heat transfer to the upper region of the PCM. As a result, even with increasing NR_t , the differences between the cases remain gradual and monotonic, with no abrupt nonlinear behaviors or strong

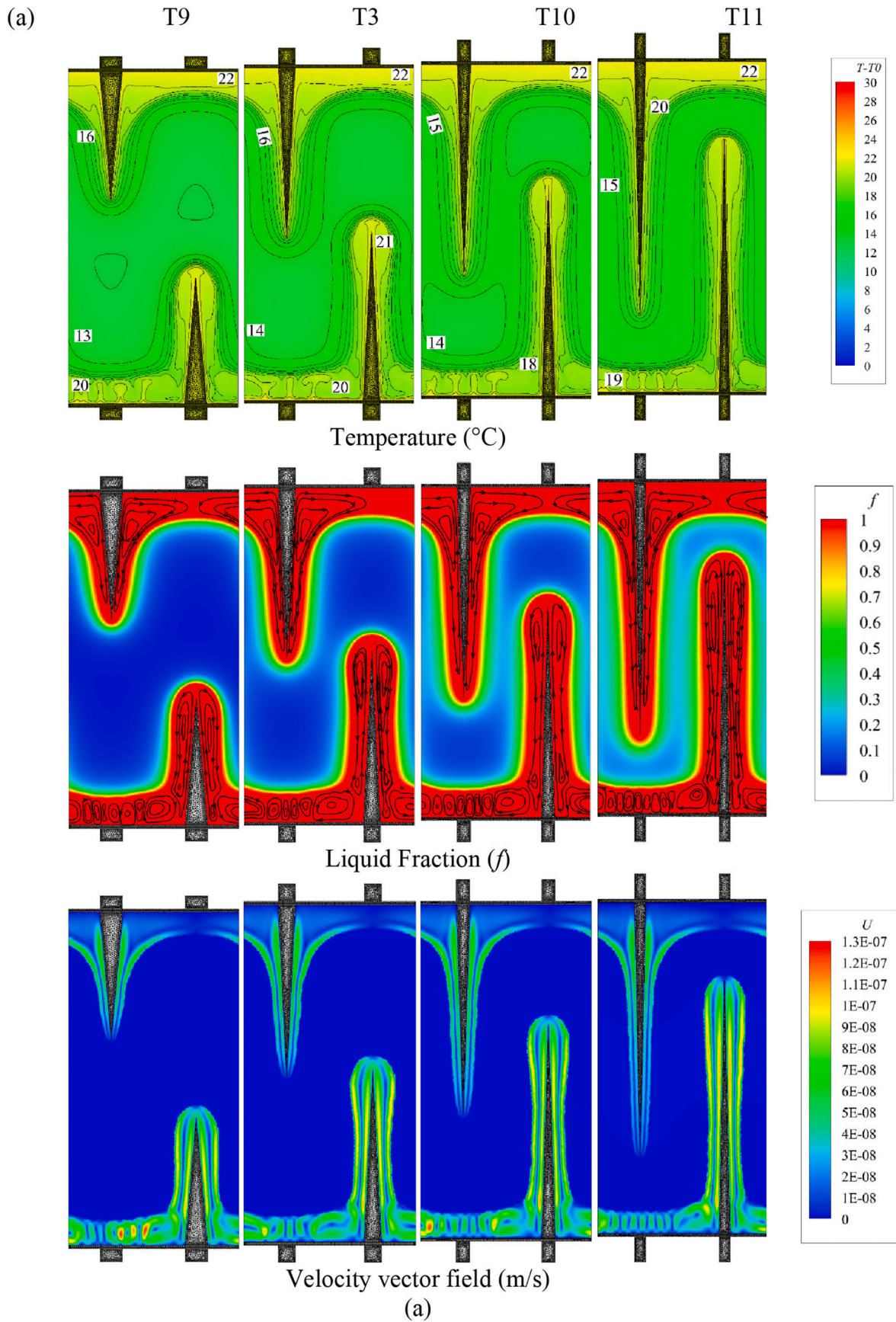


Fig. 15. Thermal and fluid fields at $t = 140$ min (melting) and $t = 150$ min (solidification) for cases T9 – T11.

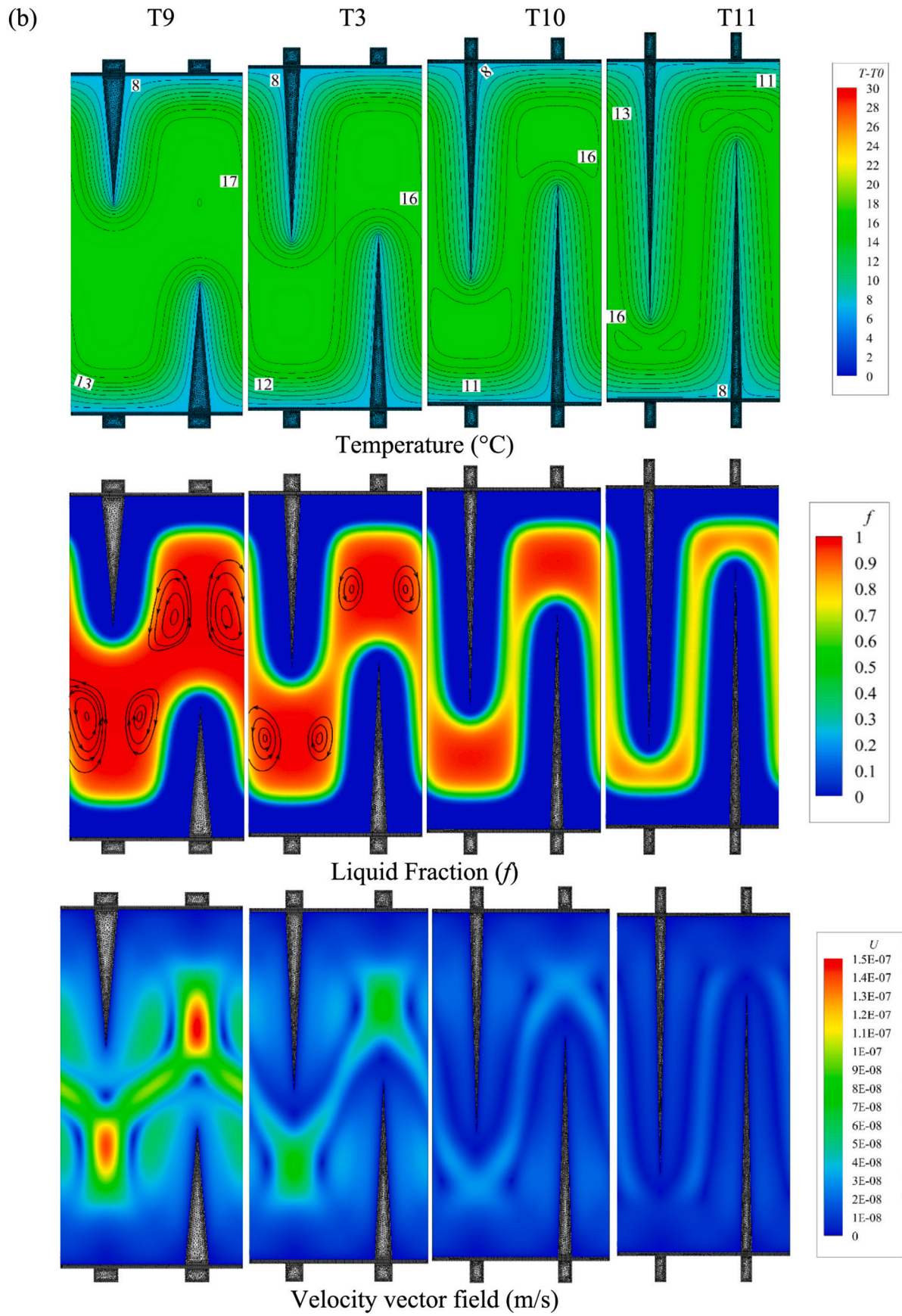


Fig. 15. (continued).

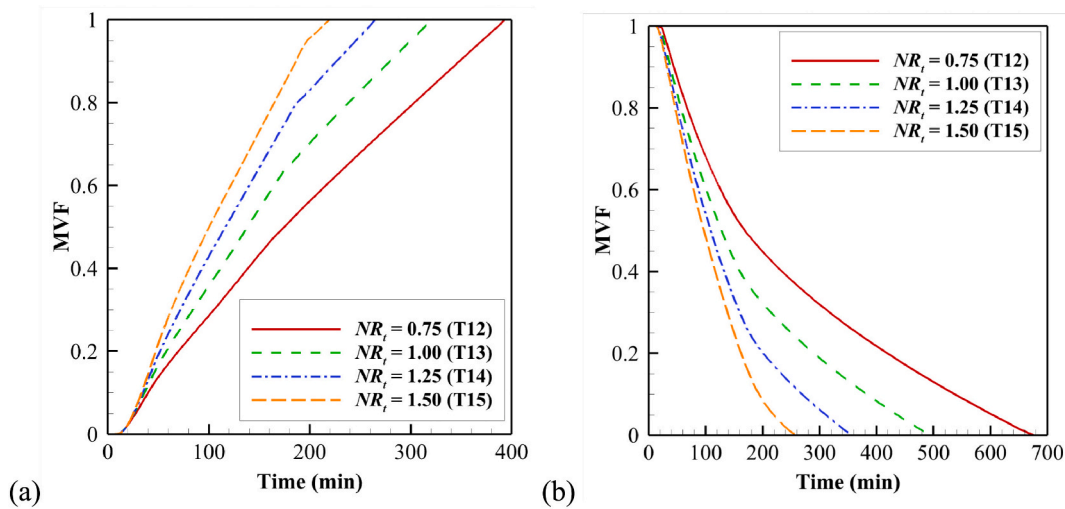


Fig. 16. *MVF* evolution for cases T12 – T15 with bottom-only fins of varying NR_f during (a) melting and (b) solidification.

performance reversals observed. The performance changes are consistent and predictable, following the expected conductive behavior based on fin reach.

Given the smooth and systematic progression of the *MVF* curves and the lack of any sudden or sharp transitions in the data, contour plots are not essential for interpreting this figure. The curves alone sufficiently demonstrate the thermal influence of fin aspect ratio under top-only configurations, which are conduction-limited by design and inherently less sensitive to fin geometry than configurations with fins penetrating deeper PCM zones.

Fig. 19 displays *MVF* evolution for cases T20 to T24, all of which include fins under insulated boundary conditions, with variations in fin height ratio NR_f from 0.1 to 0.9. Melting behavior is shown in Fig. 19 (a) and solidification in Fig. 19 (b).

Interestingly, while the curves exhibit minor separation in the melting stage (Fig. 19 (a)), the differences are subtle and only begin to slightly diverge toward the final 20–30% of the melting time. Solidification curves (Fig. 19 (b)) are even more tightly grouped, indicating minimal influence of the fin height ratio under insulated boundary conditions.

This subdued sensitivity can be explained by the absence of periodic thermal coupling, which limits cross-domain heat exchange and restricts the thermal penetration depth of the fins. Consequently, even major changes in fin proportions do not translate to strong differences in performance, as the total thermal resistance remains high in all cases.

Due to the uniformity and tight clustering of the *MVF* curves across all NR_f values, and the lack of nonlinear, abrupt, or regionally distinct thermal behavior, there is no need for additional contour plots. The thermal response trends are clear from the *MVF* curves alone, and further spatial field visualization would not yield additional insights beyond what is already evident.

4.4. Fin placement and boundary conditions

Fig. 20 presents the variation of *MVF* with time during the melting (Fig. 20 (a)) and solidification (Fig. 20 (b)) phases for five simulation cases (T3, T13, T17, T25, T26). These cases differ in terms of boundary condition type (periodic vs. insulated) and fin configuration (top-bottom fins, both top, both bottoms, or no fins).

The superiority of top-fin configurations can be explained by the coupling between fin location, buoyancy-driven motion, and thermal stratification. During melting, the liquid PCM generated near the heated copper surfaces becomes lighter and tends to rise. Therefore, the upper part of the domain becomes a thermally active region where warm liquid

accumulates and circulation is stronger. Top fins interact more directly with this active liquid layer and transfer heat into regions where buoyancy assists heat redistribution. In contrast, bottom-only fins mainly promote heat transfer in the lower part of the PCM, and the upper solid region must be reached through a longer conduction path or through weaker secondary circulation. This produces a more delayed melt-front progression in bottom-fin cases, especially when the fin penetration is limited.

During solidification, the role of natural convection is reduced as the solid layer grows and the remaining liquid becomes confined. Under these conditions, fin placement mainly affects the conductive heat-removal path. Top fins provide effective conductive bridges between the cooled boundaries and the remaining liquid/solidifying PCM in the upper and central regions, reducing the thermal resistance of the discharge process. Bottom-only fins, especially at lower tooth ratios, leave larger regions away from the fin tips and can result in persistent liquid zones, which explains their longer solidification times. Therefore, top-fin configurations perform better because they combine favorable interaction with buoyancy during melting and shorter effective conductive paths during solidification.

From the melting curves in Fig. 20 (a), it is evident that the fin configuration combined with the type of boundary condition significantly affects melting rates, especially as the process nears completion. The periodic boundary case with top-bottom fins (T3) demonstrates the fastest melting, followed closely by periodic cases with bottom (T13) and top (T17) fin configurations. In contrast, finless configurations T25 (insulated) and T26 (periodic) show delayed melting responses, with T25 performing the worst.

Similarly, in Fig. 20 (b) (solidification), T3, T13, and T17 outperform T25 and T26, especially in the later phase of the process ($MVF < 0.3$). Notably, the absence of PCM fins in T25 and T26, regardless of boundary condition, severely limits heat extraction capabilities. Interestingly, periodicity alone does not offer a meaningful advantage without extended conductive pathways.

To investigate the source of these differences, temperature, liquid fraction, and velocity vector contours are presented in Fig. 21 (a) (melting at $t = 180$ min for T3, T13, T17 and $t = 370$ min for T25, T26), and Fig. 21 (b) (solidification at the same time steps). These times correspond to the stage just before or during the onset of sharp divergence among the *MVF* curves. During melting, buoyancy-driven convection is present in the liquid PCM, but the differences among the cases are still strongly affected by fin-guided conductive penetration into the remaining solid and weak-circulation regions. During solidification, the velocity field is much weaker, so the divergence among cases is mainly

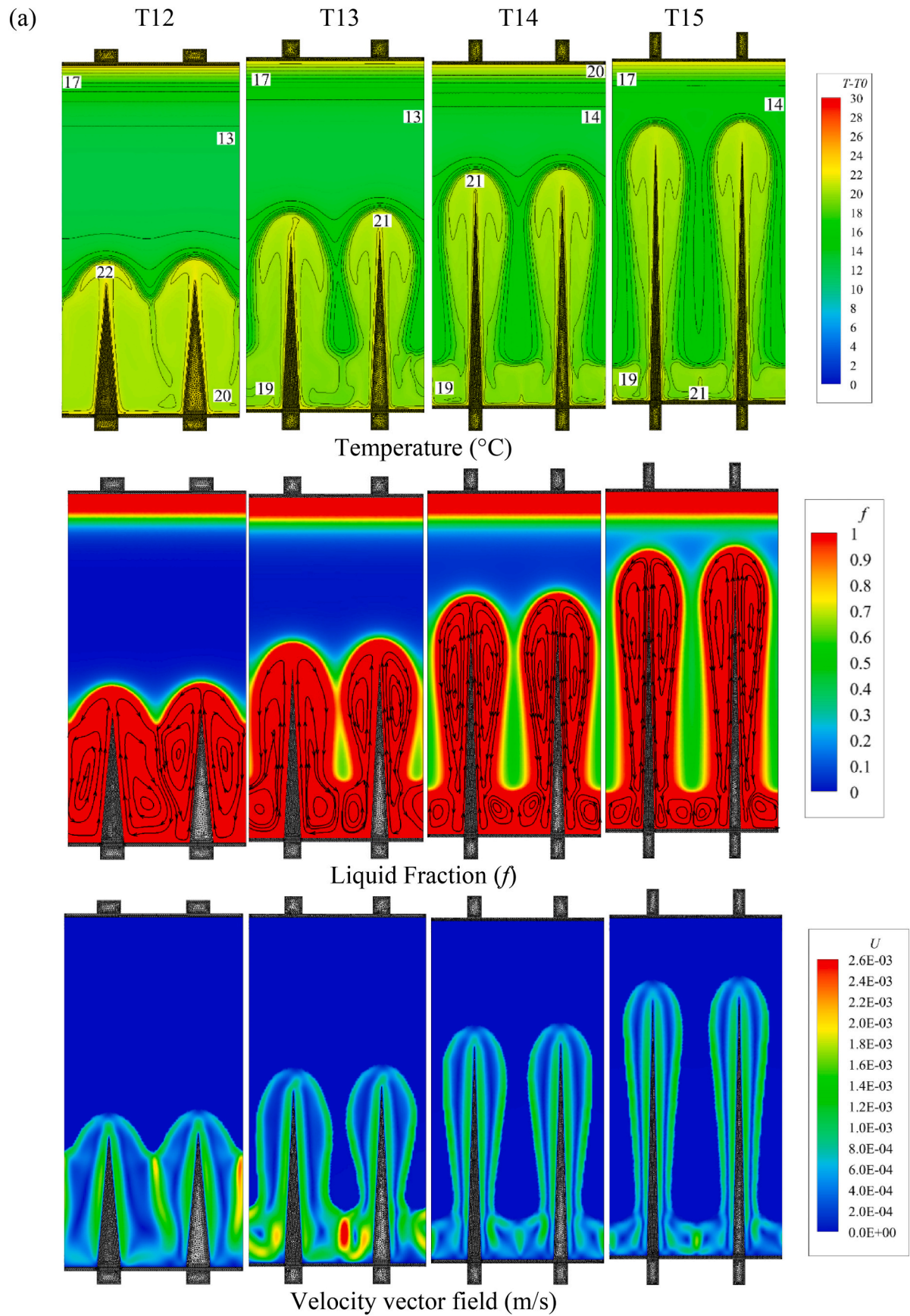


Fig. 17. Contours at $t = 170$ min (melting) and $t = 180$ (solidification) for T12 – T15.

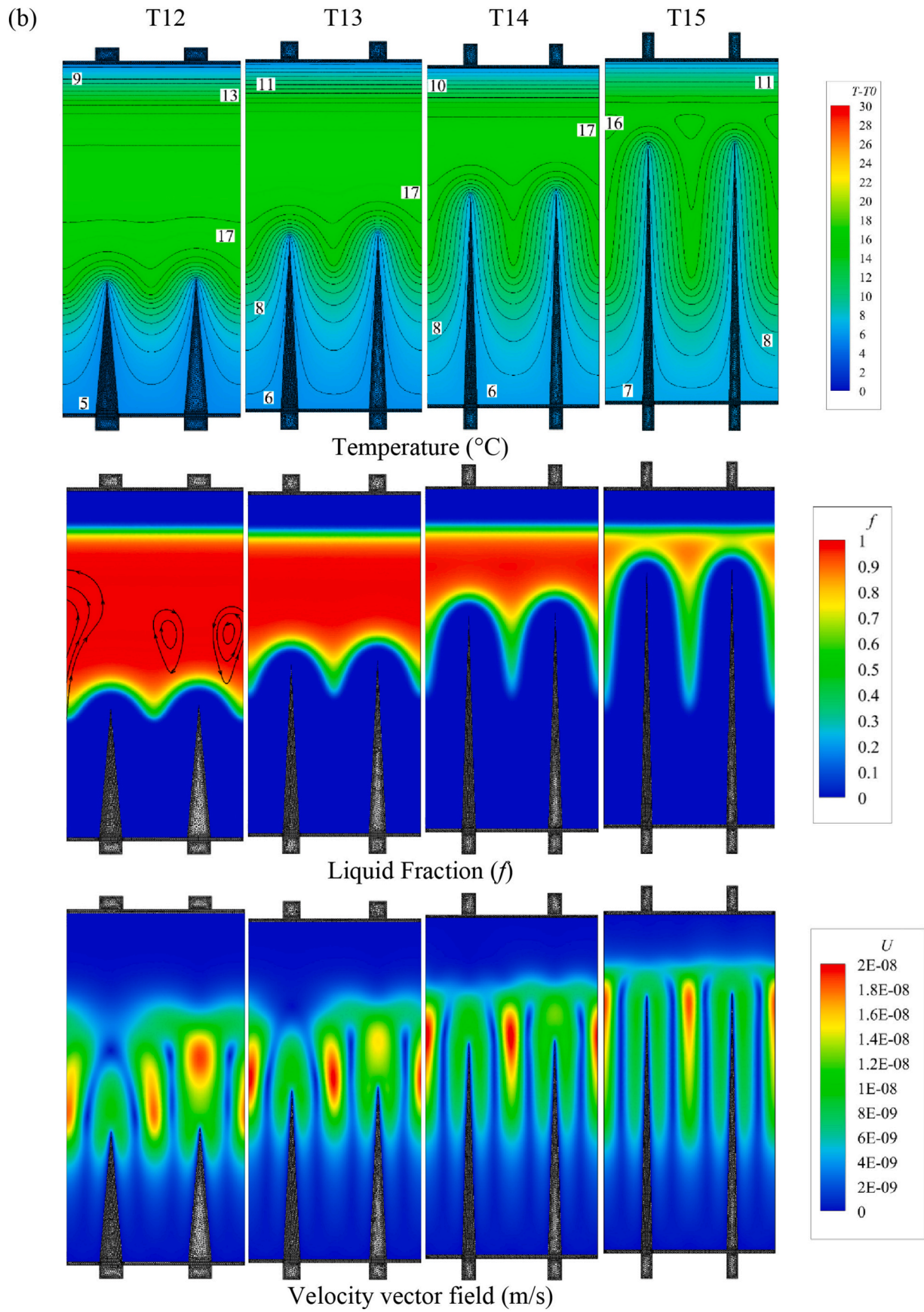


Fig. 17. (continued).

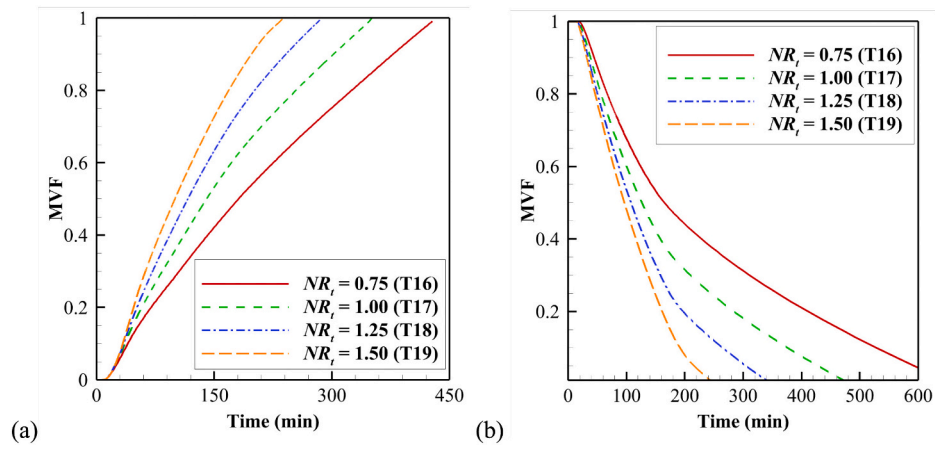


Fig. 18. MVF curves during (a) melting and (b) solidification for top-only fin configurations (T16 – T19) with varying NR_t .

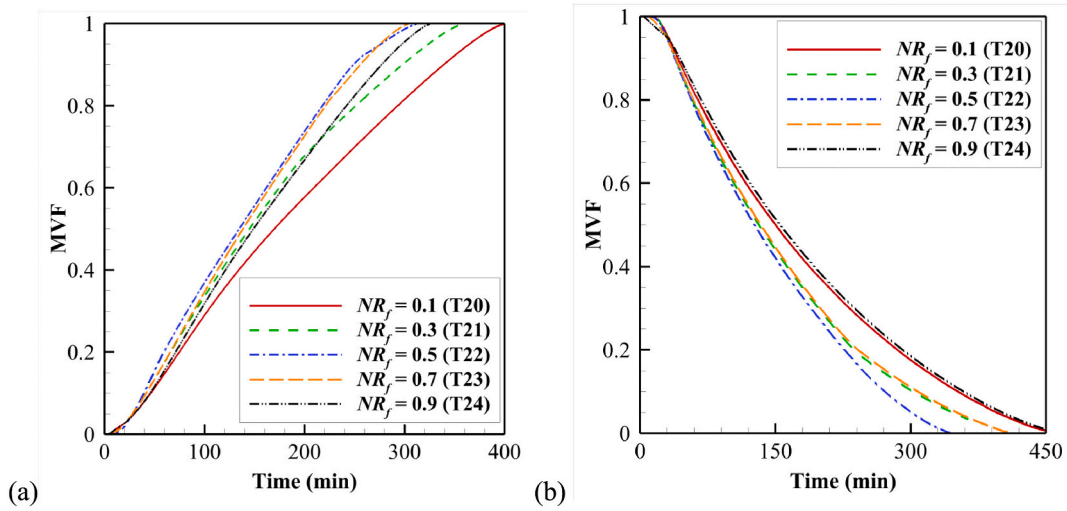


Fig. 19. MVF response for insulated boundary cases (T20 – T24) with different fin height ratios NR_f during (a) melting and (b) solidification.

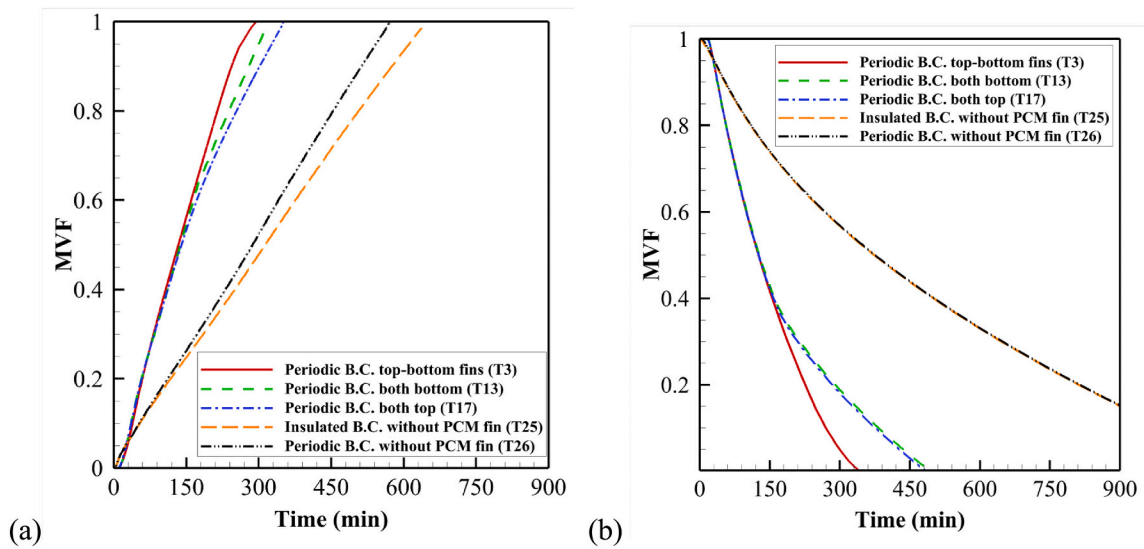


Fig. 20. Effect of boundary condition (periodic vs. insulated) and fin configuration on MVF during (a) melting, and (b) solidification.

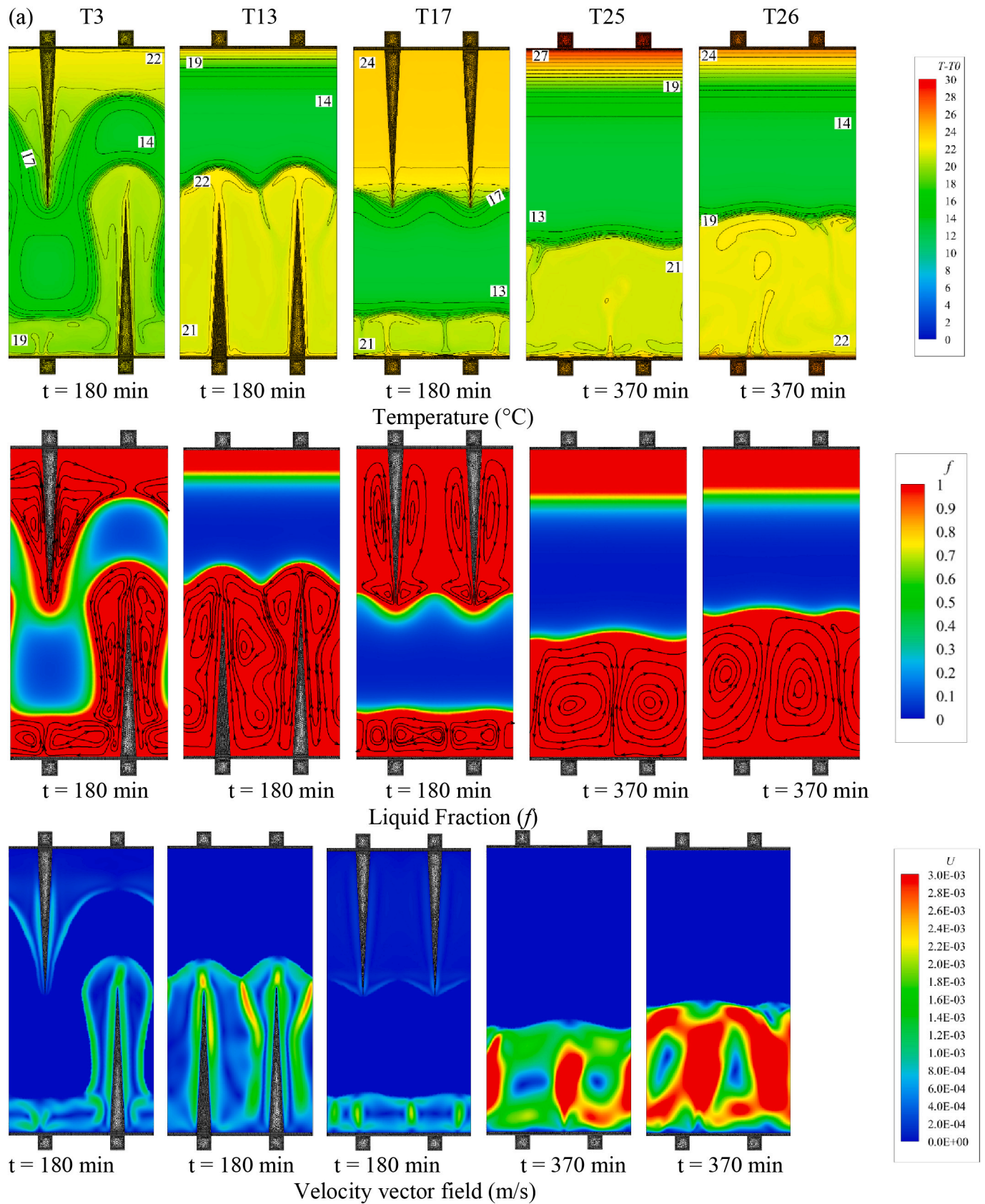
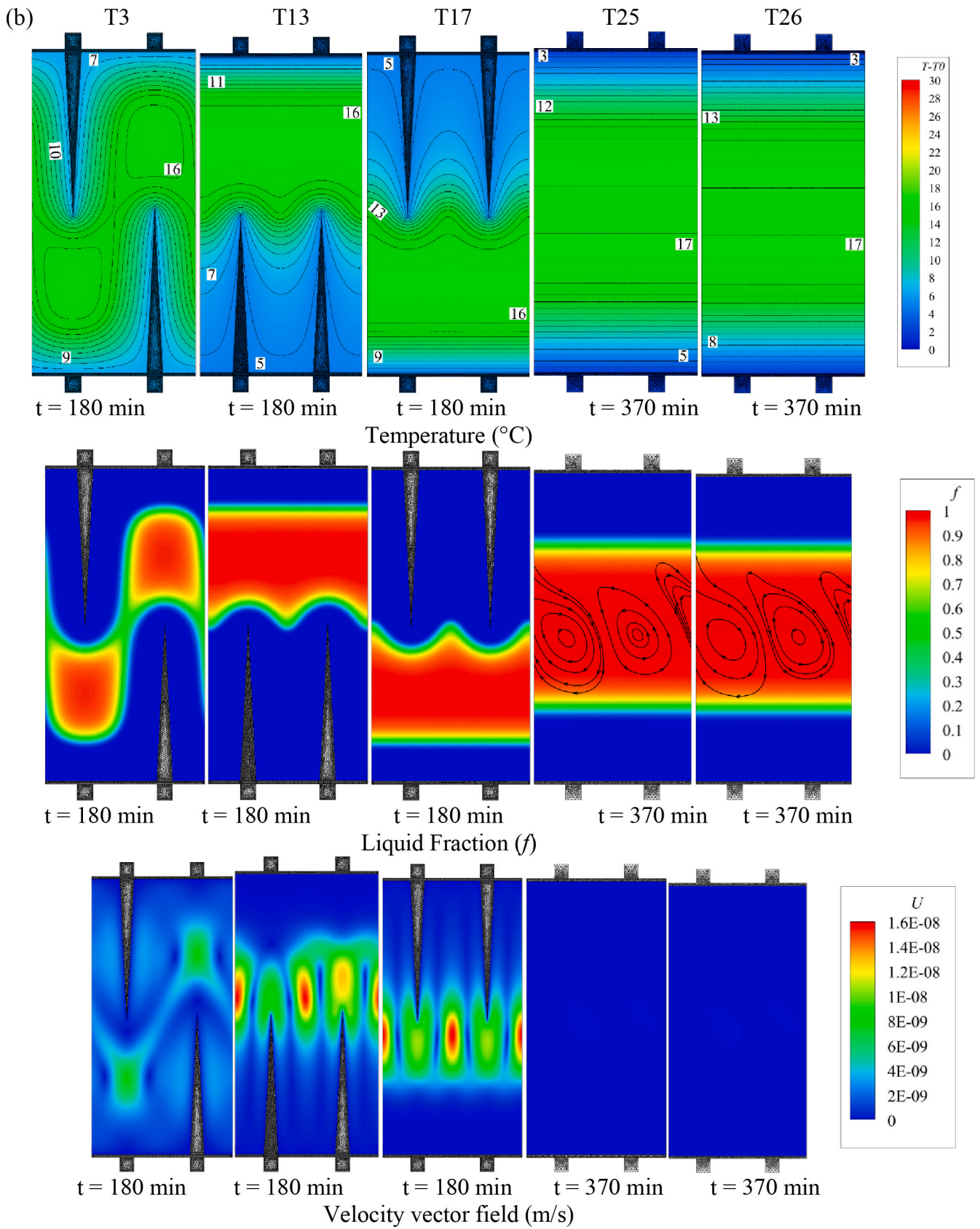


Fig. 21. Temperature difference, liquid fraction, and velocity contours at $t = 180 \text{ min}$ (T3, T13, T17) and $t = 370 \text{ min}$ (T25, T26) for (a) melting and (b) solidification.



controlled by the conductive heat-removal paths created by the fins and boundary conditions.

The temperature fields for T3, T13, and T17 (with fins) show much deeper heat penetration into the PCM domain, guided by the fin geometry. T3, in particular, exhibits strong vertical heat diffusion due to the dual fin placement, which leads to faster melting across the domain. In contrast, T25 and T26, both lacking fins, display shallow thermal gradients, with heat localized near the top, indicating limited conduction and sluggish melting (Fig. 21 (a)).

Liquid fraction contours mirror this trend: in T3 and T13, most of the PCM has already melted, while in T25 and T26, large solid regions persist at the bottom, confirming poor conductive reach. Notably, even the periodic boundary in T26 does not significantly enhance melting without conductive fin structures.

During melting, the velocity scale in Fig. 21(a) is on the order of 10^{-3} m/s, with localized maxima approaching 3×10^{-3} m/s. Thus, buoyancy-driven convection is present in the molten PCM. However, the observed differences among the cases are still strongly influenced by fin placement and conductive heat penetration into the remaining solid and weak-circulation regions.

This performance disparity can also be physically explained by examining the nature of heat diffusion paths. In cases like T3 with long and slender fins, the melted zone initially grows around the fins and extends vertically due to enhanced conduction along the finned pathways. However, small gaps of unmelted PCM tend to remain between the fins, creating a non-uniform melt front. In contrast, T13 (with short and thick fins) shows a more laterally spread but shallower molten zone with fewer gaps, forming a more continuous melt region in the lower part. This distinction becomes especially relevant when the melted height remains below the fin tips; beyond this point, the melt front becomes unified, and the geometric advantage of fins diminishes. The presence of fins near the base is particularly beneficial in the early phase of melting since natural convection initiates from bottom to top, thus emphasizing the importance of conductive structures located near the heat source.

On the other hand, during the solidification phase, temperature contours show that T3, T13, and T17 cool more rapidly and uniformly across the domain. The presence of copper fins facilitates efficient heat removal, while in T25 and T26, cooling is restricted to the upper layers, and thermal gradients persist across the PCM (Fig. 21 (b)).

Liquid fraction maps indicate that T3 and T13 are approaching near-complete solidification, whereas T25 and T26 retain high melt fractions throughout. This is consistent with their poor performance in Fig. 20 (b). The spatial progression of solidification fronts illustrates how periodic boundary conditions alone are not sufficient unless coupled with fins to bridge thermal resistance.

Finally, the velocity fields are extremely weak (below 10^{-8} m/s) in all cases, indicating that convection has not yet begun to play a meaningful role in influencing the solidification pattern. The divergence in performance among the cases is, again, attributable to thermal conduction effectiveness shaped by fin geometry and boundary conditions.

From a physical standpoint, the fin geometry and location also dictate the morphology of the solidification front. In cases with long, narrow fins (e.g., T17), solidification progresses vertically along the fins, creating tall and narrow solid regions with residual melt trapped in between. Conversely, short and thick fins (e.g., T13) yield a more laterally extended and cohesive solid zone, especially near the base. In both cases, the presence of fins near the bottom accelerates the initial phase of solidification due to the upward movement of natural convection and higher local temperature gradients. However, in configurations without fins (T25, T26), the solidification front remains nearly horizontal and advances slowly from the top down, leading to a stratified structure with persistent molten layers at the bottom. This difference confirms that fin presence is more impactful when the solidification interface lies below or within the fin height; once the interface surpasses the fin region, their influence weakens significantly.

4.5. Charging and discharging time and average power

Fig. 22 illustrates the total melting time required for each configuration to reach a MVF of 0.9. Cases with enhanced fin distribution and higher convective coefficients, such as T3, T7, and T11, exhibit significantly reduced melting times. In contrast, configurations with insulated boundary conditions or without PCM fins (e.g., T25 and T26) show delayed melting due to limited thermal diffusion. These results emphasize the role of both fin placement and boundary heat transfer in accelerating energy absorption.

Fig. 23 compares average charging power per unit depth, calculated by dividing E_{stored} by $t_{MVF=0.9}$ [Eq. (16b)]. Here E_{stored} is the increase in Q_{total} from the physical initial state to target state $MVF = 0.9$ target state. T8 produced the highest average charging power, approximately 0.112 kW/m. For the same fin geometry, this is about 85% higher than T3 (approximately 0.060 kW/m), reflecting the increase in h_{conv} from 50 to 150 $\text{W/m}^2\cdot\text{K}$. The finless baselines T25 and T26 yielded approximately 0.027 and 0.030 kW/m, respectively.

Fig. 24 displays the total solidification time, defined as the time required to reach $MVF = 0.1$. The trends parallel those in Fig. 22, though with more pronounced variation. Cases with top-bottom or bottom-only fins under periodic boundaries (e.g., T3, T11) solidify significantly faster than those with insulated conditions or no fins (e.g., T25, T26), which require up to 996 min in the evaluated cases. This disparity highlights

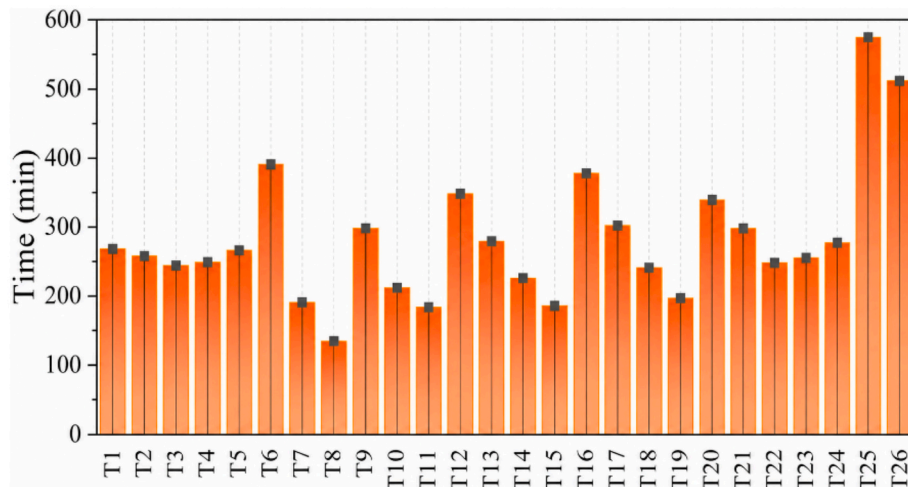


Fig. 22. Total melting time required to reach $MVF = 0.9$ for cases T1–T26.

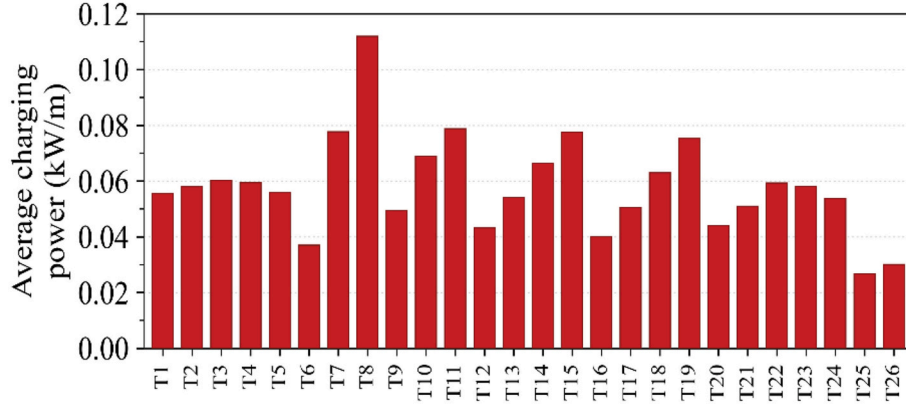


Fig. 23. Average charging power per unit depth for cases T1–T26, calculated as E_{stored} divided by $t_{\text{MVF}=0.9}$ [Eq. (16b)].

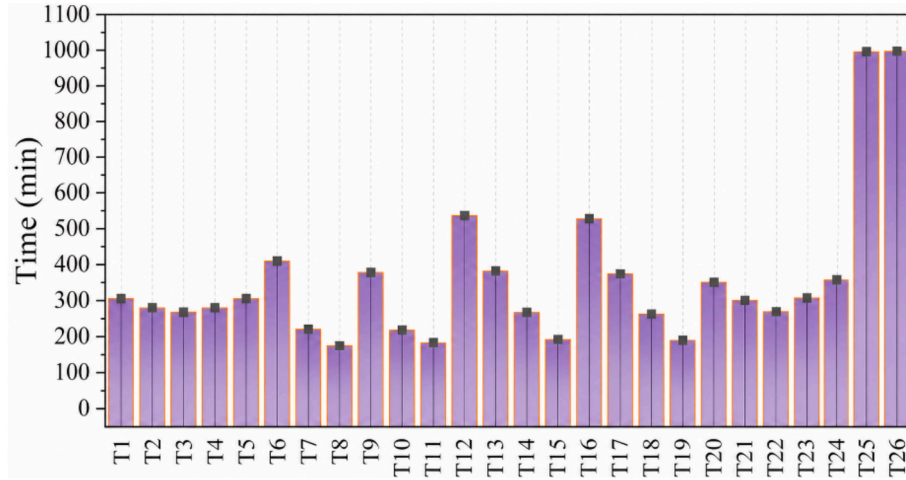


Fig. 24. Total solidification time to reach $\text{MVF} = 0.1$ for cases T1–T26.

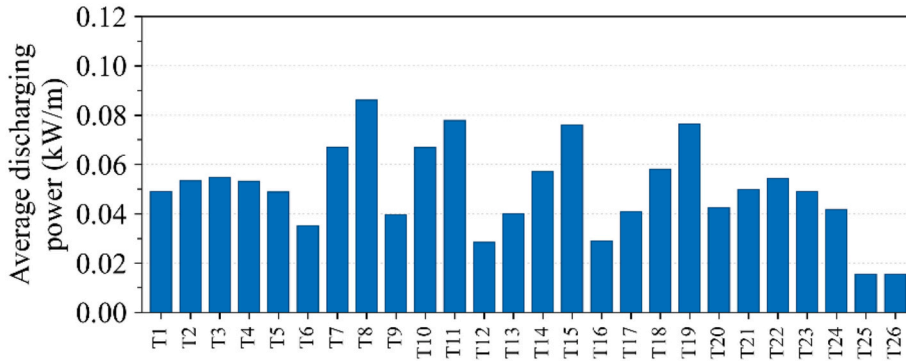


Fig. 25. Average discharging power per unit depth for cases T1–T26, calculated as E_{released} divided by $t_{\text{MVF}=0.1}$ [Eq. (16c)].

the asymmetry in charging and discharging behavior and the importance of fin geometry for thermal discharge enhancement.

Fig. 25 compares average discharging power per unit depth, calculated by dividing E_{released} by $t_{\text{MVF}=0.1}$ [Eq. (16c)]. Here E_{released} is the positive decrease in Q_{total} from the physical initial state to the target state $\text{MVF} = 0.1$ target state. T8 produced the highest average discharging power, approximately 0.086 kW/m. For the same fin geometry, this is about 57% higher than T3 (approximately 0.055 kW/m). The finless baselines T25 and T26 each yielded approximately 0.016 kW/m.

5. Conclusions

The principal findings are organized from geometry effects under matched conditions, through boundary and operating-condition effects, to system-level performance. Rate-based interpretation, model uncertainty, validation priorities, and the scope of future formal optimization are then stated separately:

- Geometry effects under matched conditions. Within constant-material finned comparisons, $NR_f = 0.5$ gave the shortest melting time among ratios 0.1–0.9 (about 8% faster than the extremes);

increasing NR_f from 0.75 to 1.5 reduced melting time by 39%; and, at matched tooth ratios, top-fin placement reduced melting time by 12.5–26.2% relative to bottom-only placement.

- Boundary and operating-condition effects. Insulated boundaries increased melting time by 1.6% for the matched $NR_f = 0.5$ pair and by up to 5% at $NR_f = 0.9$. Increasing h_{conv} from 50 to 150 W/m²·K reduced melting time by 44.9%. Because T8 uses the largest imposed h_{conv} , it is the best-performing evaluated operating case, not a jointly optimized geometry.
- System-level fin/no-fin comparison. T25 required 327% longer to reach MVF = 0.7 than T8 (440 versus 103 min); equivalently, T8 reduced melting time by 76.6%. T25 and T26 are not material-matched controls because removing the PCM-side fins reduces copper and increases PCM volume within the same envelope; they are used only as finless device baselines.
- Rate-based performance. The positive energy changes E_{stored} and $E_{released}$ were defined from Q_{total} between the physical initial state and the target state charging and discharging target states, respectively. Dividing each energy change by its corresponding elapsed time yielded average power, not energy. For the matched T3/T8 geometry, increasing h_{conv} from 50 to 150 W/m²·K increased average charging power from approximately 0.060 to 0.112 kW/m (about 85%) and average discharging power from approximately 0.055 to 0.086 kW/m (about 57%). T8 provided the highest average charging and discharging powers among the evaluated cases.
- Model uncertainty and validation. The selected NM = 6 mesh differs from NM = 8 by 0.04% in melting MVF and 0.42% in solidification MVF at $t = 250$ min, but these values quantify mesh-resolution effects only. They do not include uncertainty from the two-dimensional periodic-cell assumption, prescribed HTF-side convection, thermophysical properties, mushy-zone treatment, or extrapolation from the literature benchmark. Future work should therefore include time-step and property sensitivity, uncertainty propagation, and device-scale experimental validation across relevant boundary and operating conditions.
- Formal optimization as future work. After dedicated validation, future work should define objective functions and constraints and may then investigate machine-learning approaches, such as Physics-Informed Neural Networks (PINNs) [44,45] and metaheuristic optimization methods [46,47], within a formal multi-objective optimization of fin configurations. This would extend the present comparative study to a reproducible optimization workflow for melting and solidification performance.

CRediT authorship contribution statement

Mohammad Ghalambaz: Writing – review & editing, Writing – original draft, Visualization, Validation, Investigation, Formal analysis, Data curation, Conceptualization. **Kourosh Behzadian:** Writing – review & editing, Writing – original draft, Supervision, Investigation, Formal analysis, Data curation, Conceptualization. **Antonino La Rocca:** Writing – review & editing, Writing – original draft, Supervision, Investigation, Formal analysis, Conceptualization. **Roohollah Babaei-Mahani:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **Amir Keshmiri:** Writing – review & editing, Writing – original draft, Visualization, Software, Resources, Project administration, Funding acquisition, Formal analysis, Conceptualization. **Pouyan Talebizadehsardari:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Data curation.

Declaration of competing interest

The authors clarify that there is no conflict of interest for report.

Acknowledgement

We would like to acknowledge the funding by the UK Engineering and Physical Sciences Research Council (EPSRC – Reference: EP/X525753/1) under Impact Acceleration Account (IAA) provided to the University of Manchester, which has provided computational resources and supported some of the underpinning research presented in this paper.

Data availability

Data will be made available on request.

References

- [1] P. Arévalo, D. Ochoa-Correa, E. Villa-Ávila, Advances in thermal energy storage systems for renewable energy: a review of recent developments, *Processes* 12 (2024) 1844.
- [2] J.-T. Carnie, Y. Hardalupas, A. Sergis, Decarbonising building heating and cooling: designing a novel, inter-seasonal latent heat storage system, *Renew. Sustain. Energy Rev.* 189 (2024) 113897.
- [3] I. Sarbu, A. Dorca, Review on heat transfer analysis in thermal energy storage using latent heat storage systems and phase change materials, *Int. J. Energy Res.* 43 (2019) 29–64.
- [4] Z.L. Yang, R. Walvekar, W.P. Wong, R.K. Sharma, S. Dharaskar, M. Khalid, Advances in phase change materials, heat transfer enhancement techniques, and their applications in thermal energy storage: a comprehensive review, *J. Energy Storage* 87 (2024) 111329.
- [5] J. Bošnjak Hordov, S. Nizetić, M. Jurčević, D. Čoko, M. Čosić, M. Jakić, M. Arıcı, Review of organic and inorganic waste-based phase change composites in latent thermal energy storage: thermal properties and applications, *Energy* 306 (2024) 132421.
- [6] S. Ghalambaz, A scientometric study of nano encapsulated phase change material (NEPCM): trends and categories, *J. Nanofluids* 13 (2024) 985–998.
- [7] M.S. Kabasa, S. Biira, K.E. Kanyarusoke, Developments in the design optimization of cascaded PCM latent heat thermal energy storage units: a review, *Energy Storage Saving* 4 (2025) 439–462.
- [8] M.A. Said, H.S.S. Aljibori, J.M. Mahdi, H.I. Mohammed, P. Talebizadehsardari, A. Keshmiri, Modifying the performance kinetics in the shell-and-multi tube latent heat storage system via dedicated finned tubes for building applications, *J. Build. Eng.* 97 (2024) 110722.
- [9] H.A. Al-Salami, N.S. Dhaidan, H.H. Abbas, F.N. Al-Mousawi, R.Z. Homod, Review of PCM charging in latent heat thermal energy storage systems with fins, *Therm. Sci. Eng. Prog.* 51 (2024) 102640.
- [10] A.Q. Ahmed, S.E. Razavi, Enhancing heat transfer in L-shaped enclosures with combined natural convection-fluid structure interaction: incorporating flexible fin and elastic wall, *Numer. Heat Transf. Part A Appl.* 87 (2024). None–None.
- [11] M.E. Tiji, S. Rezaei, K. Hosseinzadeh, S. Kaplan, P. Talebizadehsardari, A. Keshmiri, Optimizing heat transfer in a finned rectangular latent heat storage system using response surface methodology, *Case Stud. Therm. Eng.* 66 (2025) 105701.
- [12] T. Abdulrazzaq, N. Biswas, T. Alsharifi, F.L. Rashid, A.F. Khalaf, A.M. Sadeq, A. E. Anqi, H. Togun, A.K. Hussein, Performance improvement of phase change material (PCM)-based shell-and-tube-type latent heat energy storage system utilizing curved fins, *J. Therm. Anal. Calorim.* 149 (2024) 14241–14255.
- [13] S.A. Shaik, P.K. Nigam, S.K. Gugulothu, Advanced fin designs for improved thermal management in PCM-based latent heat storage systems, *Appl. Therm. Eng.* 272 (2025) 126337.
- [14] B. Kamkari, H. Shokouhmand, Experimental investigation of phase change material melting in rectangular enclosures with horizontal partial fins, *Int. J. Heat Mass Transf.* 78 (2014) 839–851.
- [15] A. Abdi, M. Shahrooz, J.N.W. Chiu, V. Martin, Experimental investigation of solidification and melting in a vertically finned cavity, *Appl. Therm. Eng.* 198 (2021) 117459.
- [16] H.R. Mouziraji, M.E. Tiji, A.M. Abed, A.E. Tiji, H.I. Mohammed, J.M. Mahdi, P. Talebizadehsardari, A. Keshmiri, Boosting solidification rates in a triplex-tube thermal storage system using circular fins: novel staggered and inline fin arrangements, *Heat Transfer Eng.* 46 (2025) 1024–1045.
- [17] B. Gürel, A numerical investigation of the melting heat transfer characteristics of phase change materials in different plate heat exchanger (latent heat thermal energy storage) systems, *Int. J. Heat Mass Transf.* 148 (2020) 119117.
- [18] W. Lin, W. Zhang, Z. Ling, X. Fang, Z. Zhang, Experimental study of the thermal performance of a novel plate type heat exchanger with phase change material, *Appl. Therm. Eng.* 178 (2020) 115630.
- [19] J. Li, Y. Zhang, Z. Peng, X. Zhang, J. Zhai, Y. Luo, B. Liu, X. Sun, S.N. Al-Saadi, Thermal performance of a plate-type latent heat thermal energy storage heat exchanger - an experimental investigation and simulation study, *J. Energy Storage* 65 (2023) 107295.
- [20] M. Taghavi, A. Ferrantelli, T. Joronen, Multi-objective optimization of a plate heat exchanger thermal energy storage with phase change material, *J. Energy Storage* 89 (2024) 111645.

- [21] M. Wang, W. Liu, Thermal performance analysis of a new multiple flat-plate latent heat storage heat exchanger, *J. Energy Storage* 98 (2024) 113035.
- [22] S.K. Choi, S.O. Kim, T.H. Lee, D. Hahn, Computation of the natural convection of nanofluid in a square cavity with homogeneous and nonhomogeneous models, *Numer. Heat Transf. A Appl.* 65 (2014) 287–301.
- [23] A.I.N. Korti, H. Guellil, Experimental study of the effect of inclination angle on the paraffin melting process in a square cavity, *J. Energy Storage* 32 (2020) 101726.
- [24] A. Agarwal, R.M. Sarviya, Characterization of commercial grade paraffin wax as latent heat storage material for solar dryers, *Mater. Today Proc.* 4 (2017) 779–789.
- [25] N. Ukrainczyk, S. Kurajica, J. Sipusic, Thermophysical comparison of five commercial paraffin waxes as latent heat storage materials, *TUBILIO (Tech. Univ. Darmstadt)* 24 (2010) 129–137.
- [26] H. Zheng, C. Wang, Q. Liu, Z. Tian, X. Fan, Thermal performance of copper foam/paraffin composite phase change material, *Energ. Convers. Manage.* 157 (2018) 372–381.
- [27] S. Zhang, Y. Yao, Y. Jin, Z. Shang, Y. Yan, Heat transfer characteristics of ceramic foam/molten salt composite phase change material (CPCM) for medium-temperature thermal energy storage, *Int. J. Heat Mass Transf.* 196 (2022) 123262.
- [28] M. Ghalambaz, A.A. Melaibari, A.J. Chamkha, O. Younis, M. Sheremet, Phase change heat transfer and energy storage in a wavy-tube thermal storage unit filled with a nano-enhanced phase change material and metal foams, *J. Energy Storage* 54 (2022) 105277.
- [29] D.A. Nield, A. Bejan, *Convection in Porous Media*, Springer, New York, 2006.
- [30] F. Alimi, K. Hajlaoui, M. Bouzidi, M. Mohamed, W. Yaïci, N.H. Alrasheedi, M. Ghalambaz, P. Talebidadehsardari, Charging and discharging heat transfer enhancement in a latent thermal energy storage array using petal-shaped tubes and fins, *Case Stud. Therm. Eng.* 74 (2025) 106917.
- [31] J. Shafi, O. Younis, S. Tiari, M. Ghalambaz, Artificial intelligence–numerical study of melting and solidification heat transfer in a bundle of petal tubes embedded in metal foam, *Appl. Therm. Eng.* 279 (2025) 127960.
- [32] M. Boujelbene, S.A. Mehryan, M. Sheremet, M. Shahabadi, N.B. Elbashir, M. Ghalambaz, Numerical study of a non-Newtonian phase change flow in finned rectangular enclosures, *Facta Univ. Ser. Mech. Eng.* 23 (2024) 287–310.
- [33] M. Bouzidi, Z. Raizah, B.S. Chauhan, A. Hajjar, T. Anwar, M. Mohamed, H. Khmissi, M. Ghalambaz, Dynamic melting with a flow of phase change material over an electrical element for fast latent heat thermal energy storage cooling, *Appl. Therm. Eng.* 289 (2026) 129808.
- [34] C.T. Kelley, *Solving Nonlinear Equations with Newton's Method*, SIAM, 2003.
- [35] P. Deuffhard, *Newton Methods for Nonlinear Problems: Affine Invariance and Adaptive Algorithms*, Springer Science & Business Media, 2011.
- [36] O. Schenk, K. Gärtner, Solving unsymmetric sparse systems of linear equations with PARDISO, *Future Gener. Comput. Syst.* 20 (2004) 475–487.
- [37] Intel Corporation, oneMKL PARDISO-parallel direct sparse solver interface, in: *Developer Reference for Intel® oneAPI Math Kernel Library for Fortran*, Intel Corporation, Santa Clara, CA, 2024.
- [38] D.M. Pedroso, Caveats of three direct linear solvers for finite element analyses, *Int. J. Numer. Methods Eng.* 125 (2024) e7545.
- [39] B. Kamkari, H. Shokouhmand, F. Bruno, Experimental investigation of the effect of inclination angle on convection-driven melting of phase change material in a rectangular enclosure, *Int. J. Heat Mass Transf.* 72 (2014) 186–200.
- [40] B. Kamkari, H.J. Amlashi, Numerical simulation and experimental verification of constrained melting of phase change material in inclined rectangular enclosures, *Int. Commun. Heat Mass Transf.* 88 (2017) 211–219.
- [41] A. Abdi, M. Shahrooz, J.N. Chiu, V. Martin, Experimental investigation of solidification and melting in a vertically finned cavity, *Appl. Therm. Eng.* 198 (2021) 117459.
- [42] K. Teather, K. Siddiqui, An experimental investigation of constrained melting of a phase change material (PCM) in circular geometries, part I: velocity characterization, *J. Energy Storage* 86 (2024) 111516.
- [43] K. Teather, K. Siddiqui, An experimental investigation of solidification of a Phase Change Material (PCM) in circular enclosures, *Int. J. Therm. Sci.* 220 (2026) 110282.
- [44] D. Jalili, M. Jadidi, A. Keshmiri, B. Chakraborty, A. Georgoulas, Y. Mahmoudi, Transfer learning through physics-informed neural networks for bubble growth in superheated liquid domains, *Int. J. Heat Mass Transfer* 232 (2024) 125940.
- [45] D. Jalili, S. Jang, M. Jadidi, G. Giustini, A. Keshmiri, Y. Mahmoudi, Physics-informed neural networks for heat transfer prediction in two-phase flows, *Int. J. Heat Mass Transf.* 221 (2024) 125089.
- [46] M. Ghalambaz, R.J. Yengejeh, A.H. Davami, Building energy optimization using grey wolf optimizer (GWO), *Case Stud. Therm. Eng.* 27 (2021) 101250.
- [47] F. Alimi, N.H. Alrasheedi, M. Edalatfar, J. Shafi, K. Hajlaoui, S. Chebaane, M. Bouzidi, M. Ghalambaz, Artificial intelligence-computational fluid dynamics study close contact melting of phase change materials in a finned shell-tube energy storage, *Case Stud. Therm. Eng.* 75 (2025) 107015.